[11-09-08-L]

5.5 Exercises

Multiply, then simplify the products. Assume that all variables represent positive real numbers. See Example 1.

1. $3(5 - \sqrt{6})$
2. $2(\sqrt{3} - \sqrt{7})$
3. $\sqrt{6}(3 + \sqrt{2})$
4. $\sqrt{2}(\sqrt{32} - \sqrt{9})$
5. $5(\sqrt{72} - \sqrt{8})$
6. $\sqrt{3}(\sqrt{12} + 2)$
7. $\sqrt{5}(\sqrt{15} - \sqrt{5})$
8. $\sqrt{2}(\sqrt{18} + \sqrt{3})$
9. $(\sqrt{7} + 3)(\sqrt{7} - 3)$
10. $(\sqrt{3} - 5)(\sqrt{3} + 5)$
11. $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$
12. $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})$
13. $(\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2})$
14. $(\sqrt{20} - \sqrt{5})(\sqrt{20} + \sqrt{5})$
15. $(\sqrt{2} + 1)(\sqrt{3} - 1)$
16. $(\sqrt{3} + 3)(\sqrt{5} - 2)$
17. $(\sqrt{11} - \sqrt{7})(\sqrt{2} + \sqrt{5})$
18. $(\sqrt{6} + \sqrt{2})(\sqrt{3} + \sqrt{2})$
19. $(\sqrt{7} + \sqrt{5})(\sqrt{6} - \sqrt{2})$
20. $(\sqrt{3} - \sqrt{2})(\sqrt{5} - \sqrt{6})$
21. $(2\sqrt{3} + \sqrt{5})(3\sqrt{3} - 2\sqrt{5})$
22. $(\sqrt{7} - \sqrt{11})(2\sqrt{7} + 3\sqrt{11})$
23. $(\sqrt{5} + 2)^2$
24. $(\sqrt{11} - 1)^2$
25. $(\sqrt{21} - \sqrt{5})^2$
26. $(\sqrt{6} - \sqrt{2})^2$
27. $(\sqrt{8} + \sqrt{2})^2$
28. $(\sqrt{27} - \sqrt{3})^2$
29. $(2 + \sqrt[3]{6})(2 - \sqrt[3]{6})$
30. $(\sqrt[3]{3} + 6)(\sqrt[3]{3} - 6)$
31. $(2 + \sqrt[3]{2})(4 - 2\sqrt[3]{2} + \sqrt[3]{4})$
32. $(\sqrt[3]{3} - 1)(\sqrt[3]{9} + \sqrt[3]{3} + 1)$
33. $(3\sqrt{x} - \sqrt{5})(2\sqrt{x} + 1)$
34. $(4\sqrt{p} + \sqrt{7})(\sqrt{p} - 9)$
35. $(3\sqrt{r} - \sqrt{s})(3\sqrt{r} + \sqrt{s})$
36. $(\sqrt{k} + 4\sqrt{m})(\sqrt{k} - 4\sqrt{m})$
37. $(5\sqrt{z} + 1)^2$
38. $(6\sqrt{a} - 5)^2$
39. $(\sqrt[3]{2y} - 5)(4\sqrt[3]{2y} + 1)$
40. $(\sqrt[3]{9z} - 2)(5\sqrt[3]{9z} + 7)$

Complete each factorization. Assume that all variables represent positive real numbers.

41. $2\sqrt{3} + 2 = 2(\quad)$
42. $3 - 3\sqrt{2} = 3(\quad)$
43. $3\sqrt{2} + 3\sqrt{5} = 3(\quad)$
44. $5\sqrt{3} + 5\sqrt{7} = 5(\quad)$
45. $2\sqrt{20} - 4\sqrt{7} = 4(\quad)$
46. $6\sqrt{5} - \sqrt{8} = 2(\quad)$
47. $5x + \sqrt{50x^3} = 5x(\quad)$
48. $12p^3 - \sqrt{72p^5} = 6p^2(\quad)$

Write each expression in lowest terms. Assume that all variables represent positive real numbers. See Example 2.

49. $\dfrac{30 - 20\sqrt{6}}{10}$
50. $\dfrac{15 - 6\sqrt{5}}{12}$
51. $\dfrac{3 - 3\sqrt{5}}{3}$
52. $\dfrac{-5 + 5\sqrt{2}}{5}$
53. $\dfrac{16 - 4\sqrt{8}}{12}$
54. $\dfrac{12 - 9\sqrt{72}}{18}$
55. $\dfrac{6p - \sqrt{24p^3}}{3p}$
56. $\dfrac{11y - \sqrt{242y^5}}{22y}$

Rationalize the denominators in each of the following. Assume that all variables represent positive real numbers. See Example 3.

57. $\dfrac{3}{4 + \sqrt{5}}$
58. $\dfrac{4}{3 - \sqrt{7}}$
59. $\dfrac{6}{\sqrt{5} - 1}$
60. $\dfrac{8}{\sqrt{5} + 3}$
61. $\dfrac{2}{\sqrt{2} + \sqrt{5}}$
62. $\dfrac{-5}{\sqrt{3} - \sqrt{7}}$
63. $\dfrac{\sqrt{8}}{3 - \sqrt{2}}$
64. $\dfrac{\sqrt{27}}{2 + \sqrt{3}}$
65. $\dfrac{1}{2 - 3\sqrt{2}}$
66. $\dfrac{5}{2\sqrt{3} + 1}$
67. $\dfrac{2}{3\sqrt{5} + 2\sqrt{3}}$
68. $\dfrac{-1}{3\sqrt{2} - 2\sqrt{7}}$

69. $\dfrac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}$ **70.** $\dfrac{\sqrt{3}+1}{\sqrt{5}+\sqrt{3}}$ **71.** $\dfrac{2-\sqrt{3}}{\sqrt{6}-\sqrt{5}}$ **72.** $\dfrac{5+\sqrt{6}}{\sqrt{3}-\sqrt{2}}$

73. $\dfrac{m-4}{\sqrt{m}+2}$ **74.** $\dfrac{r-9}{\sqrt{r}-3}$ **75.** $\dfrac{3\sqrt{x}}{\sqrt{x}-2\sqrt{y}}$ **76.** $\dfrac{5\sqrt{k}}{2\sqrt{k}+\sqrt{q}}$

77. $\dfrac{\sqrt{m}-\sqrt{3r}}{\sqrt{m}+\sqrt{3r}}$ **78.** $\dfrac{3\sqrt{2p}+\sqrt{5s}}{\sqrt{2p}-3\sqrt{5s}}$

In calculus, it is sometimes necessary to rationalize the numerator *of a radical expression. Rationalize the numerator in each of the following. Assume that all variables represent positive real numbers.*

79. $\dfrac{\sqrt{3}}{2}$ **80.** $\dfrac{\sqrt{11}}{5}$ **81.** $\dfrac{6-\sqrt{2}}{4}$ **82.** $\dfrac{8\sqrt{5}-1}{6}$

83. $\dfrac{\sqrt{5}+\sqrt{7}}{3}$ **84.** $\dfrac{\sqrt{15}-\sqrt{6}}{4}$ **85.** $\dfrac{3\sqrt{a}+\sqrt{b}}{b}$ **86.** $\dfrac{2\sqrt{p}-3\sqrt{q}}{4q}$

Show that the following are true.

87. $(\sqrt[3]{5}-\sqrt[3]{3})(\sqrt[3]{25}+\sqrt[3]{15}+\sqrt[3]{9})=2$ **88.** $(\sqrt[3]{y}+\sqrt[3]{2})(\sqrt[3]{y^2}-\sqrt[3]{2y}+\sqrt[3]{4})=y+2$

Use the results of Exercises 87 and 88 to rationalize the denominators of the following.

89. $\dfrac{2}{\sqrt[3]{5}-\sqrt[3]{3}}$ **90.** $\dfrac{-3}{\sqrt[3]{y}+\sqrt[3]{2}}$

Review Exercises *Solve each equation. See Sections 2.1 and 3.9.*

91. $8y-7=9$ **92.** $-2r+3=5$ **93.** $p^2-5p+4=0$ **94.** $y^2+3y+2=0$

95. $6m^2=7m+3$ **96.** $15a^2+2=11a$

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1. $15-3\sqrt{6}$ **3.** $3\sqrt{6}+2\sqrt{3}$ **5.** $20\sqrt{2}$ **7.** $5\sqrt{3}-5$ **9.** -2 **11.** -1 **13.** 6 **15.** $\sqrt{6}-\sqrt{2}+\sqrt{3}-1$ **17.** $\sqrt{22}+\sqrt{55}-\sqrt{14}-\sqrt{35}$ **19.** $\sqrt{42}-\sqrt{14}+\sqrt{30}-\sqrt{10}$ **21.** $8-\sqrt{15}$ **23.** $9+4\sqrt{5}$ **25.** $26-2\sqrt{105}$ **27.** 18 **29.** $4-\sqrt[3]{36}$ **31.** 10 **33.** $6x+3\sqrt{x}-2\sqrt{5x}-\sqrt{5}$ **35.** $9r-s$ **37.** $25z+10\sqrt{z}+1$ **39.** $4\sqrt[3]{4y^2}-19\sqrt[3]{2y}-5$ **41.** $2(\sqrt{3}+1)$ **43.** $3(\sqrt{2}+\sqrt{5})$ **45.** $4(\sqrt{5}-\sqrt{7})$ **47.** $5x(1+\sqrt{2x})$ **49.** $3-2\sqrt{6}$ **51.** $1-\sqrt{5}$ **53.** $(4-2\sqrt{2})/3$ **55.** $(6-2\sqrt{6p})/3$ **57.** $3(4-\sqrt{5})/11$ **59.** $3(\sqrt{5}+1)/2$ **61.** $(-2\sqrt{2}+2\sqrt{5})/3$ **63.** $(6\sqrt{2}+4)/7$ **65.** $-(2+3\sqrt{2})/14$ **67.** $(6\sqrt{5}-4\sqrt{3})/33$ or $2(3\sqrt{5}-2\sqrt{3})/33$ **69.** $-2+\sqrt{6}+\sqrt{2}-\sqrt{3}$ **71.** $2\sqrt{6}+2\sqrt{5}-3\sqrt{2}-\sqrt{15}$ **73.** $\sqrt{m}-2$ **75.** $(3x+6\sqrt{xy})/(x-4y)$ **77.** $(m-2\sqrt{3mr}+3r)/(m-3r)$ **79.** $3/(2\sqrt{3})$ **81.** $17/[2(6+\sqrt{2})]$ **83.** $-2/[3(\sqrt{5}-\sqrt{7})]$ **85.** $(9a-b)/[b(3\sqrt{a}-\sqrt{b})]$ **89.** $\sqrt[3]{25}+\sqrt[3]{15}+\sqrt[3]{9}$ **91.** $\{2\}$ **93.** $\{1, 4\}$ **95.** $\{-1/3, 3/2\}$

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