

■ Sometimes it helps to use the progression of differences repeatedly. Example below.

Find the general term of $\{a_n\} = 1, 2, 5, 12, 25, 46, 77, \dots$

$$\{a_n\} = 1, 2, 5, 12, 25, 46, 77, \dots$$

$$\{b_n\} = 1, 3, 7, 13, 21, 31, \dots$$

$$\{c_n\} = 2, 4, 6, 8, 10, \dots, 2k, \dots$$

Repeated use of progression of differences.

$$b_n = 1 + \sum_{k=1}^{n-1} 2k = n^2 - n + 1$$

$$a_n = 1 + \sum_{k=1}^{n-1} (k^2 - k + 1) = \frac{n^3}{3} - n^2 + \frac{5n}{3} = \frac{n^3 - 3n^2 + 5n}{3}$$

Is this correct?

$$\text{In}[63]:= \text{Table}\left[\frac{n^3 - 3n^2 + 5n}{3}, \{n, 1, 7\}\right]$$

$$\text{Out}[63]= \{1, 2, 5, 12, 25, 46, 77\}$$

It appears correct.

What if one still cannot see a general term in $\{c_n\}$? Well, form $\{d_n\} = 2, 2, 2, 2$. Can't miss it now!