Homework: http://math.mnrt.net/

Name _____Date_____ ____ raw ____ scaled ____ percent

Math 11 Trimester 3 Exam 1P (165 Points)

Complex numbers and applied trigonometry

- **Answers must be simplified. No denominators may include radicals. All fractions reduced. Simple arithmetic must be completely performed; e.g. write 9 instead of $\sqrt{81}$, $2\sqrt{5}$ instead of $\sqrt{20}$, 125 instead of 5^3.**
- **All angles you write for answers must be written with respect to the angle zero (the positive horizontal axis) and measured in the positive direction (counter clockwise). For example, write $\theta = \frac{3\pi}{2}$ rather than $\theta = -\frac{\pi}{2}$.**
- **Calculators are allowed, but only <u>exact</u> answers will receive credit, unless a question says otherwise.**
- **All arguments of complex numbers in polar form must be between 0 and 2π.**
- **Each question counts 15 points.**

[1] Write z in trigonometric form $r(\cos\theta + i\sin\theta)$ with θ in radians, when $z = 3\sqrt{3} - 3i$.

[2] Write z in Euler form using radians, when $z = -1 - i$.

[3] Write z in rectangular form, when $z = e^{(\frac{7\pi}{6})i}$.

[4] Write z in Euler form using degrees to the nearest $\frac{1}{10}$ °, when $z = 3 - 4i$.

[5] Let $z = (3 + 2i)$ and $w = 1 - i$. Find the product $z\,w$.

[6] Let $z = 3\,\text{cis}(\frac{7\pi}{8})$ and $w = 2\,\text{cis}(\frac{2\pi}{11})$. Find the product $z\,w$.

[7] Find $\left(\sqrt{3} + i\right)^5$ and write the answer in rectangular form.

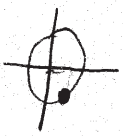
[8] Find $\left(\sqrt{2}\,e^{\frac{\pi}{4}i}\right)^8$ and write the answer rectangular form.

[9] Find $[2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})]^6$ and write the answer in rectangular form.

[10] Solve in the complex numbers: $x^3 - 8 = 0$ and write answer in rectangular from.

[11] A ship is steaming at 12 miles per hour on a straight course from point M to point N. When the ship is at S_1, the navigator sights a lighthouse L and finds that $\angle LS_1N = 30°$. One hour later the ship is at S_2 and $\angle LS_2N = 70°$. Find the distance of the lighthouse from the line of the ship's course. (Answer accurate to 2 significant figures.)

[1] Write z in trigonometric form $r(\cos\theta + i\sin\theta)$ with θ in radians, when $z = 3\sqrt{3} - 3i$.



4 BUT IN III

$3\sqrt{3} - 3i$

$r = \sqrt{(3\sqrt{3})^2 + 3^2} = \sqrt{27+9} = \sqrt{36} = 6$

ok, but how did this morph into 3 here?

$\sin\theta = -\frac{3}{6} = -\frac{1}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$3\sqrt{3} - 3i = 3\left[\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right]$

$3\sqrt{3} - 3i$

$r = \sqrt{(3\sqrt{3})^2 + 3^2} = \sqrt{27+9} = \sqrt{36} = 6$

$\tan\theta \quad \frac{-3}{3\sqrt{3}} = -\frac{1}{\sqrt{3}} = \frac{11\pi}{6}$

$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

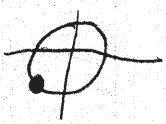
$6\left[\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right]$

copy error

[2] Write z in Euler form using radians, when $z = -1 - i$.

$re^{\theta i}$

$-1-1$



Good

$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\cos\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} = \frac{5\pi}{4}$

$\sqrt{2}e^{\left(\frac{5\pi}{4}\right)i} = -1-1$

[3] Write z in rectangular form, when $z = e^{(\frac{7\pi}{6})i}$.

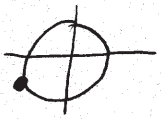
$\frac{7\pi}{6}i$

$1\,e$

$a = r\cos\theta$

$b = r\sin\theta$

$1\,\mathrm{Cis}\,\frac{7\pi}{6}$



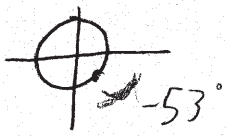
$\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}$

$-\frac{\sqrt{3}}{2} - i\frac{1}{2}$

[4] Write z in Euler form using degrees to the nearest $\frac{1}{10}$ °, when $z = 3 - 4i$.

$z = 3-4i$

$r = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$



$\sin^{-1}\left(\frac{-4}{5}\right) = -53.1$

$5[\cos(-53.1) + i\sin(-53.1)]$

$5\text{cis}(-53.1)$

$5e^{-53.1°i}$

positives pls.

[5] Let $z = (3 + 2\,i)$ and $w = 1 - i$. Find the product $z\,w$.

$(3+2i)(1-i)$

$3+2i-3i-2i^2$

$3-i+2$

$\boxed{5-i}$

[6] Let $z = 3\,\mathrm{cis}(\frac{7\pi}{8})$ and $w = 2\,\mathrm{cis}(\frac{2\pi}{11})$. Find the product $z\,w$.

$(3\,\mathrm{cis}(\frac{7\pi}{8}))(2\,\mathrm{cis}(\frac{2\pi}{11}))$

$6\,\mathrm{cis}(\frac{7\pi}{8} + \frac{2\pi}{11})$

$6\,\mathrm{cis}(\frac{93\pi}{88})$

$\frac{7\pi}{8} = \frac{77\pi}{88}$

$\frac{2\pi}{11} = \frac{16\pi}{88}$

[7] Find $\left(\sqrt{3}+i\right)^5$ and write the answer in rectangular form.

$\sqrt{3}+i$

$r=\sqrt{(\sqrt{3})^2+1^2}=\sqrt{3+1}=2$

$\tan\theta=\frac{1}{\sqrt{3}}=\frac{\pi}{6}$

$\left[2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]^5$

$2^5\left(\cos(5)\left(\frac{\pi}{6}\right)+i\sin(5)\left(\frac{\pi}{6}\right)\right)$

$32\left[\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right]$

$32\left[-\frac{\sqrt{3}}{2}+i\frac{1}{2}\right]$

$-16\sqrt{3}+16i$

[8] Find $\left(\sqrt{2}\, e^{\frac{\pi}{4} i}\right)^8$ and write the answer rectangular form.

$\left(\sqrt{2}\, \text{cis}\, \frac{\pi}{4}\right)^8$

$\left(\sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]\right)^8$

$\left(\sqrt{2}\right)^8\left[\cos 8\left(\frac{\pi}{4}\right) + i\sin(8)\left(\frac{\pi}{4}\right)\right]$

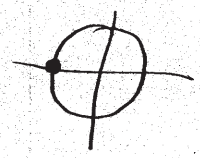
$16\left[\cos 2\pi + i\sin 2\pi\right]$

$16\left[\cos 0 + i\sin 0\right]$

$16\left[1 + 0\right]$

$\boxed{16}$

[9] Find $[2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})]^6$



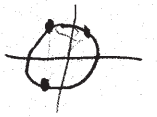
$2^6(\cos(6)(\frac{\pi}{6}) + i\sin(6)(\frac{\pi}{6}))$

$*(64(\cos\pi + i\sin\pi))*$

$64(-1) + 64(0)$

$\boxed{-64}$

[10] Solve in the complex numbers: $x^3 - 8 = 0$ and write answer in rectangular from..

$x^3 = 8$ $\quad 8[\cos 0 + i\sin 0]$  } nice touch!

$w_0 = 8^{\frac{1}{3}}\left[\cos\frac{0+0}{3} + i\sin\frac{0+0}{3}\right] = 2[\cos 0 + i\sin 0] = 2$ ✓

$w_1 = 8^{\frac{1}{3}}\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right] = 2\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right] = -1 + i\sqrt{3}$ ✓ 

$w_2 = 2\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right] = 2\left[-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right] = -1 - i\sqrt{3}$ ✓ 

$\frac{2}{1}\cdot\frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

Excellent!