# [11-09-13-T11 - REV]

*Function*

This is a somewhat formal treatment of the idea of a function. There are many informal characterizations of a function. For example, a function is said to be a rule that shows how one number may be unambiguously computed given a number. Another characterization of a function is that it is like a machine that receives an input and produces an unambiguous output. These informal characterizations of a function are important. And they have there place in the historical development of the concept of function. They will not be treated here, but it is hoped that you are familiar with them from previous courses in mathematics.

In the this discussion of the idea of a function we take a function to be a relation between the elements of two sets that pairs no more than one element of the second set, called the **range** of the function with each element of the first set called the **domain** of the function. An element of the domain is referred to as an argument of the function and the corresponding element of the range is called the value of the function at that argument.

We could define a function the be a collection of ordered pairs each of whose first components is an element of the domain and each of whose second components is an element of the range such that no second component appears more than once. Then the collection $\{(-2, 5), (3, 9), (4, 8)\}$ is a function but the collection $\{(-2, 5), (3, 9), (4, 8), (10, 5)\}$ is not a function.

We will instead use the following definition.

DEFINITION FUNCTION. Let $x_1$ and $x_2$ be elements of a set $\mathcal{A}$ and let $y_1$ and $y_2$ be elements of another set $\mathcal{B}$. Let R be a relation that pairs $x_1$ with $y_1$ and $x_2$ with $y_2$. If $x_1 = x_2 \Longrightarrow y_1 = y_2$, then the relation R is a function whose domain is the set $\mathcal{A}$ and whose range is the set $\mathcal{B}$.

Notation. Often, functions are named by lower case letters f, g, h. We write $y = f(x)$, pronounced "y is a function of x", where the item parenthesis, in this example $x$, is the argument of the function and $y$ is the value of the function at the argument $x$.

It is important to note that a function is identified by specifying both the relation and the domain. Changing the specification of the domain, changes the function.

DEFINITION INVERSE FUNCTION. Let $f$ be a function that pairs $x_1$ with $y_1$ and $x_2$ with $y_2$. If $y_1 = y_2 \Longrightarrow x_1 = x_2$, then there exists another function called the inverse function of $f$, often denoted by $f^{-1}$, with the property that $f \circ f^{-1} = I = f^{-1} \circ f$, where $I : I(x) = x$.

DEFINITION INCREASING FUNCTION. Let $f$ be a function that pairs $u$ with $v$, so that $(u, v) \in f$. If on an interval I contained in the domain of $f$, $v > u \Longrightarrow f(v) > f(u)$ for every $u, v \in I$, then $f$ is increasing on I.