Exercises [A]

[09-01-06-RVL11]

1. Write the domain and range of the functions defined below.
   (a) $f = \{(1, 4), (2, 9), (3, 19), (4, 51)\}$ (b) $f(x) = x^3$ (c) $f(x) = x^4 + 2$
   (d) $f(x) = \dfrac{6}{x-2}$ (e) $f(x) = \sqrt{x^2 - 4}$

2. (a) Explain why $y^2 = 9 - x^2$ does not define a function.
   (b) Does $y = \sqrt{9 - x^2}$ define a function?

3. State the domain of the function defined by each equation.
   (a) $y = \dfrac{4}{x(x-2)}$ (b) $f(x) = \dfrac{x}{x^2 - 1}$ (c) $y = \dfrac{2}{\sqrt{x-1}}$

4. If $f(x) = 4\sqrt{x-1} + x + 5$, find the least value of f.
   Hint: First find the domain of f.

5. $f(x) = 4x - 3$. Write the simplest form of the following expressions.
   (a) $f(3)$ (b) $f(0)$ (c) $f(-2)$ (d) $f(n)$ (e) $f(x^2)$
   (f) $f(n+1) - f(n)$ (g) $\dfrac{f(a) - f(b)}{a - b}$ (h) $\dfrac{f(x_1 + h) - f(x_1)}{h}$

6. $f(x) = 9 - 3x$. Write the simplest form of the following expressions.
   (a) $f(0)$ (b) $f(4)$ (c) $f(-3)$ (d) $f(2k)$ (e) $f(x+3)$
   (f) $f(n+1) - f(n)$ (g) $\dfrac{f(a) - f(b)}{a - b}$ (h) $\dfrac{f(x_1 + h) - f(x_1)}{h}$

7. $f(x) = x^4 - 4x^2$. Show that $f(-x_1) = f(x_1)$.
   (b) What can be deduced about the symmetry of the graph of f?

8. (a) $f(x) = x^3 - 4x$. Show that $f(-x_1) = -f(x_1)$.
   (b) What can be deduced about the symmetry of the graph of f?

9. Which of the following statements are true for every function f?
   (a) The graph of f is symmetric about the y-axis.
   (b) The graph of f is symmetric about the x-axis.
   (c) If $x_1 = x_2$, then $f(x_1) = f(x_2)$.
   (d) If $f(x_1) = f(x_2)$, then $x_1 = x_2$.
   (e) A vertical line meets the graph of f in no more than one point.
   (f) A horizontal line meets the graph of f in no more than one point.

10. Show that if a linear function is defined by $f(x) = mx + k$, the expression $\dfrac{f(a) - f(b)}{a - b}$ is the slope of the graph of f when a, b are elements in the domain of f and $a \neq b$.

11. $f(x) = x^2$. Write the simplest form of the following expressions.
   (a) $f(3) - f(-3)$ (b) $f(x_1) - f(-x_1)$
   (c) $f(a+b) - f(a-b)$ (d) $\dfrac{f(x_1 + h) - f(x_1)}{h}$

12. $f(x) = x^2$. Evaluate $\dfrac{f(a) - f(b)}{a - b}$, where $a \neq b$, and give a geometric interpretation of the result.

Exercises [A]

1. $f(x) = 6 - 3x$. (a) State the zero of f. (b) Find the solution set of $f(x) > 0$. (c) Find the change in $f(x)$ when x is increased by 4.
2. $f(x) = 2x + 5$. (a) State the zero of f. (b) Find the solution set of $f(x) > 0$. (c) Find the change in $f(x)$ when x is increased by 7.
3. A function F is defined by the rule $y = 2x$ with the domain the set of positive integers. What is the range of F?
4. A function g is defined by the rule $y = \frac{1}{2}x$ with the range the set of positive integers. What is the domain of g?
5. $f(x) = 3x + 4$ and $g(x) = 10 - x$. (a) Find $f \cap g$. (b) Find the solution set of $f(x) \geq g(x)$.
6. $f(x) = 3x + 4$ and $g(x) = 3x + 8$. (a) Find $f \cap g$. (b) Find the solution set of $g(x) > f(x)$.
7. The number of units of length of a metal bar is given by the formula $l = 100.000 + 0.003(t - 55)$, when t is the temperature on the Fahrenheit scale. (a) Find l when $t = 55$. (b) Find l when $t = 0$. (c) Find the least integer t for which $l > 100.140$. (d) Find the increase in l corresponding to an increase in temperature of 20°.
8. (a) $f(x) = 12 - 3x$. Evaluate $f(4) - f(1)$.
   (b) $f(x) = 2.38x + 15.72$. Evaluate $f(17.5) - f(7.5)$.
   (c) $f(x) = mx + k$. Write the simplest expression for $f(n + 3) - f(n)$.
9. (a) If $f(x) = 2x - 5$, evaluate $f(a)$, $f(b)$, $f(a + b)$. (b) Is it true that $f(a + b) = f(a) + f(b)$?
10. (a) If $f(x) = 4.2x$, evaluate $f(a)$, $f(b)$, $f(a + b)$. (b) Is it true that $f(a + b) = f(a) + f(b)$?
11. If $f(x) = mx + k$, under what condition does $f(a + b) = f(a) + f(b)$ for all a and b?
12. If F is a linear function which contains $(2, -3)$ and $(0, 1)$, find $F(x)$.
13. If g is a linear function such that $g(0) = 4$ and $g(4) = 0$, find $g(t)$.
14. If f is a linear function such that $f(-2) = 4$ and $\Delta f(x) = 3 \cdot \Delta x$, find $f(x)$.
15. If $f(t) = 3t - 2$, find (a) $f(2x)$, (b) $f(x^2)$, (c) $f(2t + 1)$.
16. If $f(t) = 3t - 2$, find x in terms of t so that $f(x) = t$.

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1. Domain (a) $\{1, 2, 3, 4\}$ (b) $\{x: x \in R\}$ (c) $\{x: x \in R\}$
   (d) $\{x: x \neq 2, x \in R\}$ (e) $\{x: |x| \leq 2, x \in R\}$
   Range (a) $\{4, 9, 19, 51\}$ (b) $\{f(x): f(x) \in R\}$
   (c) $\{f(x): f(x) \geq 2, f(x) \in R\}$ (d) $\{f(x): f(x) \neq 0, f(x) \in R\}$
   (e) $\{f(x): f(x) \geq 0, f(x) \in R\}$
3. (a) $\{x: x \neq 0, x \neq 2, x \in R\}$ (b) $\{x: x \neq 1, x \neq -1, x \in R\}$
   (c) $\{x: x > 1, x \in R\}$
5. (a) 9 (b) -3 (c) -11 (d) $4n - 3$ (e) $4x^2 - 3$ (f) 4 (g) 4 (h) 4
9. (a) False (b) False (c) True (d) False (e) True (f) False
11. (a) 0 (b) 0 (c) $4ab$ (d) $2x_1 + h$

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1. (a) $f(2)$ (b) $x < 2$ (c) -12 3. even positive integers
5. (a) $\{\frac{3}{2}, \frac{17}{2}\}$ (b) $x \geq \frac{3}{2}$ 7. (a) 100,000 (b) 99.835 (c) 102 (d) 0.06
9. (a) $2a - 5$; $2b - 5$; $2a + 2b - 5$ (b) No 13. $-t + 4$
15. (a) $6x - 2$ (b) $3x^2 - 2$ (c) $6t + 1$