P95

[1.1] $x^2 - 4x > 0 \Rightarrow x(x-4) > 0 \Rightarrow x \in (-\infty, 4) \cup (4, \infty+)$

[1.2] $x^2 - 8x + 12 \leq 0 \Rightarrow (x-2)(x-6) \leq 0 \Rightarrow x \in [2, 6]$

[1.3] $2x^2 - 3x - 2 < 0 \Rightarrow (x-2)(x+\frac{1}{2}) < 0 \Rightarrow x \in (-\frac{1}{2}, 2)$

[1.4] $3x^2 - 7x + 10 \geqslant 2x^2$

$x^2 - 7x + 10 \geqslant 0$

$(x-2)(x-5) \geqslant 0$

$x \in (-\infty, 2] \cup [5, \infty+)$

[1.5] $(x+1)(x+4) > 18$

$x^2 + 5x + 4 > 18$

$x^2 + 5x - 14 > 0$

$(x+7)(x-2) > 0$

$x \in (-\infty, -7) \cup (2, \infty+)$

[1.6] $\frac{5x^2}{2} - \frac{x}{6} \geqslant \frac{1}{3}$

$15x^2 - x \geqslant 2$

$15x^2 - x - 2 \geqslant 0$

$x \in (-\infty, -\frac{1}{3}] \cup [\frac{2}{5}, \infty+)$

P97

[2.1] $x^2 - 4x - 1 > 0$

$x = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$

$x \in (-\infty, 2-\sqrt{5}) \cup (2+\sqrt{5}, \infty+)$

$\rightarrow$

P97, ctd

[2.2]

$$2x^2 - 6x + 3 < 0$$

$$x = \frac{3 \pm \sqrt{9-6}}{2} = \frac{3 \pm \sqrt{3}}{2}$$

$$x \in \left(\frac{3-\sqrt{3}}{2}, \frac{3+\sqrt{3}}{2} \right)$$

[2.3]

$$-5x^2 - 3x + 1 \leq 0$$

$$5x^2 + 3x - 1 \geq 0$$

$$x = \frac{-3 \pm \sqrt{9+20}}{10} = \frac{-3 \pm \sqrt{29}}{10}$$

$$x \in \left(-\infty, \frac{-3-\sqrt{29}}{10}\right) \cup \left(\frac{-3+\sqrt{29}}{10}, \infty+\right)$$

[2.4]

$$2x^2 + 8x + 5 \leq 0$$

$$x = \frac{-4 \pm \sqrt{16-10}}{2} = \frac{-4 \pm \sqrt{6}}{2}$$

$$x \in \left[\frac{-4-\sqrt{6}}{2}, \frac{-4+\sqrt{6}}{2} \right]$$

[3] Roots are $-3, 2$, so $-p = -3+2 = -1$
$q = (-3)(2) = -6$

then

$$x^2 + px + q > 0 \equiv x^2 + x - 6 > 0$$

$$\therefore\ p = 1,\ q = -6$$

P98

[4,1] $x^2+4x+4>0$

$(x+2)^2>0$

$\therefore\ x \in \mathbb{R}-\{-2\}$ or $x \neq -2$

[4,2] $4x^2-4x+1<0$

$$x=\frac{2\pm\sqrt{4-4}}{4}=\frac{1}{2}$$

$\therefore\ x \in \phi$

[4,3] $9x^2-12x+4 \geqslant 0$

$$x=\frac{6\pm\sqrt{36-36}}{9}$$

$\therefore\ x \in \mathbb{R}$

[4,4] $64 \leq 16x-x^2$

$x^2-16x+64 \leq 0$

$$x=\frac{8\pm\sqrt{64-64}}{1}=8$$

$\therefore\ x=8$

p99

[5.1] $x^2 - x + 1 > 0$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$\therefore\ x \in \mathbb{R}$

[5.2] $x^2 + 2x + 5 < 0$

$$x = \frac{-2 \sqrt{4-5}}{}$$

$\therefore\ x \in \phi$

[5.3] $2x^2 - 4x + 3 \geqslant 0$

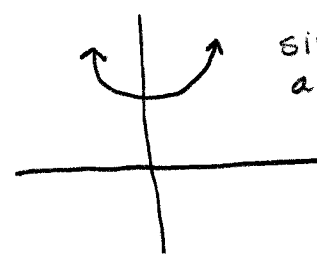
$$x = \frac{2 \pm \sqrt{4-6}}{}$$, since $D < 0$, and $a = 2$ is pos
all soln lie above $y = 0$.

$\therefore\ x \in \mathbb{R}$

[5.4] $5x \geqslant x^2 + 9$

$x^2 - 5x + 9 \leq 0$

$b^2 - 4ac = 25 - 36$,



since $a > 0$ and $D < 0$

$\therefore\ \phi$

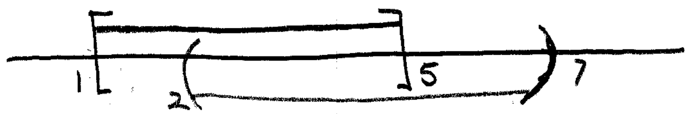
P100

[6] $\begin{bmatrix} x^2-6x+5 \le 0 \\ x^2-9x+14 < 0 \end{bmatrix}$

$x^2-6x+5 \le 0$ AND $x^2-9x+14<0$

$(x-1)(x-5) \le 0$ | $(x-7)(x-2)<0$

$x \in [1,5]$ | $x \in (2,7)$



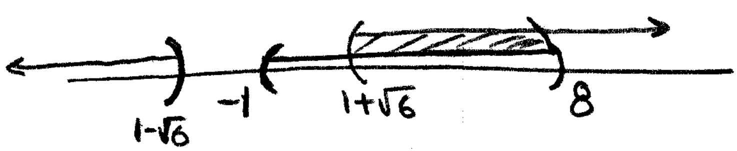
$\boxed{\therefore\ x \in (2,5]}$

[7] Find range $x \ni$ $2x+5 < x^2 < 8+7x$

$2x+5 < x^2$ AND $x^2 < 8+7x$

$x^2-2x-5>0$ | $x^2-7x-8<0$

$x \in (-\infty, 1-\sqrt{6}) \cup (1+\sqrt{6}, \infty+)$ | $x \in (-1,8)$



$\therefore\ x \in (1+\sqrt{6}, 8)$

$\longrightarrow$

[8] get $x \in \mathbb{Z} \ni$

$x^2 - 4x + 2 > 0$ AND $x^2 + 2x - 15 < 0$

$x \in (-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty+)$
AND $x \in$ Integer, so

$x \in \{\ldots, -3, -2, -1, 0, 4, 5, 6, \ldots\}$

$x \in (-5, 3)$ AND $x \in \mathbb{Z}$,
so

$x \in \{-4, -3, -2, -1, 0, 1, 2\}$

$\therefore \quad x \in \{-4, -3, -2, -1, 0\}$

NOTE this is a <u>set</u> of integers Not an interval of $\mathbb{R}$.

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[9] Find $p \in \mathbb{R}$ s.t

$$x^2 + \underbrace{2(2p+1)}_{\text{EVEN}} x \underbrace{- (p^2-1)} = 0$$ has complex solns.

$b' = 2p+1 \qquad c = -(p^2-1)$

$D < 0$ iff complex SOLNS

$$(2p+1)^2 + (p^2-1) < 0$$
$$4p^2 + 4p + 1 + p^2 - 1 < 0$$
$$5p^2 + 4p < 0$$
$$p(5p+4) < 0$$

So $p \in (-\frac{4}{5}, 0)$