J10 PPS 59-63

P 60

[1] $\begin{bmatrix} 3x+2y=6 \\ -3y=9 \end{bmatrix} \Rightarrow y=-3,\quad \begin{aligned} 3x-6&=6 \\ 3x&=12 \\ x&=4 \end{aligned}$

$\therefore\ x=4,\ y=-3$

P61

[2.1] $\alpha = 2+3i$
$\beta = 1-5i$

$\alpha+\beta = 3-2i$

[2.2] $\alpha-\beta = 1+8i$

[2.3] $\alpha\beta = (2+3i)(1-5i) = 18-7i$

[2.4] $\dfrac{\alpha}{\beta} = \dfrac{(2+3i)}{(1-5i)} \cdot \dfrac{(1+5i)}{(1+5i)} = \dfrac{-13+8i}{26} = -\frac{1}{2} + \frac{4}{13}i$

DEMO 2

P62

[2.1] $7+3i$

[2.2] $3-3i$

[2.3] 40

[2.4] $\dfrac{-3+2i}{2+3i} \cdot \dfrac{2-3i}{2-3i} = \dfrac{-2i}{13}$

i	i
i^2	-1
i^3	$-i$
i^4	1

[3.1] $i^3 = i^2 \cdot i = -i$

[3.2] 1

[3.3] $\dfrac{1}{i} \cdot \dfrac{-i}{-i} = \dfrac{-i}{-i^2} = \dfrac{-i}{1} = -i$

[3.4] $3i(1+2i)^2 = 3i(-3+4i) = -9i+12i^2 = -12-9i$

[3.5] $= [(1-i)^2]^2 = [2-2i]^2 = 4(1-i)^2 = 8-8i$

[3.6]

[3.6] $\frac{1+2i}{3-i} + \frac{1-2i}{3+i} = \frac{(1+2i)(3+i)}{(3-i)(3+i)} + \frac{(1-2i)(3-i)}{(3-i)(3+i)}$

$= \frac{1+6i+1-7i}{10}$

$= \frac{2-i}{10} = \frac{1}{5} - \frac{1}{10}i$

[4.1] $6-4i$

[4.2] $2+7i$

[4.3] $-\sqrt{2}\,i$

[4.4] -5

[5] Let β be complex conjugate of $\alpha = a+bi$, $a, b \in \mathbb{R}$

Then $\beta = a-bi$.

$\alpha + \beta = a+bi+a-bi$

$= 2a + 0i$

Since $a \in \mathbb{R}$ and $\mathbb{R}$ closed under mult, $2a \in \mathbb{R}$

$\alpha\beta = (a+bi)(a-bi)$

$= a^2 + abi - abi - b^2 i^2$

$= a^2 - b^2 i^2$

$= a^2 - b^2(-1)$

$= a^2 + b^2$

$\in \mathbb{R}$.

P64

[6.1] $\sqrt{-8} = 2i\sqrt{2}$

[6.2] $-i\sqrt{2}$

[6.3] $\frac{1}{4} i\sqrt{7}$

[7.1] $i\sqrt{48} - i\sqrt{12} = 2i\sqrt{12} - i\sqrt{12} = i\sqrt{12}$

[7.2] $i\sqrt{28} \cdot i\sqrt{35} = 2i\sqrt{7} \cdot i\sqrt{35} = -2\sqrt{7 \cdot 35} = -14\sqrt{5}$

[7.3] $\frac{3i}{3} = i$

[7.4] $\frac{3}{i} = -3i$

we usually do not leave i in denom.

$\frac{3}{i} = \frac{3i}{i^2} = -3i$

[7.5] $(3 + i\sqrt{2})(3 - i\sqrt{2}) = 9 + 2 = 11$

P65

[8.1] $\sqrt{-2}\,\sqrt{-5} = i\sqrt{2}\, i\sqrt{5} = -\sqrt{10}$

$\sqrt{(-2)(-5)} = \sqrt{10}$

opposite in sign.

[8.2] $\sqrt{2}\,\sqrt{-5} = i\sqrt{10}$

$\sqrt{2(-5)} = i\sqrt{10}$

SAME

[8.3] $\frac{\sqrt{-2}}{\sqrt{-5}} = \frac{i\sqrt{2}}{i\sqrt{5}} = \sqrt{\frac{2}{5}}$

$\sqrt{\frac{-2}{-5}} = \sqrt{\frac{2}{5}}$

SAME

[8.4] $\frac{\sqrt{2}}{\sqrt{-5}} = \frac{\sqrt{2}}{i\sqrt{5}} = \underbrace{-\frac{i\sqrt{10}}{5}, \quad \sqrt{\frac{2}{-5}} = i\sqrt{\frac{2}{5}} = \frac{i\sqrt{10}}{5}}_{\text{OPPOSITE SIGN}}$

p 67

[1.1] $\left\{\left\{x \to \frac{1}{2}\left(-3-\sqrt{13}\right)\right\}, \left\{x \to \frac{1}{2}\left(-3+\sqrt{13}\right)\right\}\right\}$

[1.2] $\{\{x \to \frac{1}{3}\}, \{x \to 2\}\}$

[1.3] $\left\{\left\{x \to \frac{1}{5}\left(3-i\sqrt{11}\right)\right\}, \left\{x \to \frac{1}{5}\left(3+i\sqrt{11}\right)\right\}\right\}$

[1.4] $\left\{\left\{x \to \frac{1}{16}\left(-13-i\sqrt{23}\right)\right\}, \left\{x \to \frac{1}{16}\left(-13+i\sqrt{23}\right)\right\}\right\}$

[1.5] $\{\{x \to 0\}, \{x \to \frac{3}{2}\}\}$

[1.6] $\{\{x \to \frac{3}{2} - \frac{3i}{2}\}, \{x \to \frac{3}{2} + \frac{3i}{2}\}\}$

[2] $a x^2 + (2 b') x + c = 0 \Longleftrightarrow x = \frac{-2 b' \pm \sqrt{4 (b')^2 - 4 a c}}{2 a} = \frac{-2 b' \pm 2 \sqrt{(b')^2 - a c}}{2 a} = \frac{- b' \pm \sqrt{(b')^2 - a c}}{a}$

[3.1] $x = \frac{3 \pm \sqrt{(-3)^2 + 3}}{1} = 3 \pm 2\sqrt{3}$

[3.2] $x = \frac{-6 \pm \sqrt{(6)^2 - (9)(7)}}{9} = \frac{1}{3}\left(-2 \pm i\sqrt{3}\right)$

p. 69

[4.1] $D = 0$, so one solution multiplicity two

[4.2] $D > 0$, two non-equal real solutions

[4.3] $D > 0$, two non-equal real solutions

[4.4] $D < 0$, two non-equal imaginary solutions

[5]
$a x^2 + b x + c = 0 \Longleftrightarrow D = b^2 - 4 a c$. b^2 is positive for all real numbers. If $a c < 0$, the $-4 a c > 0$. Then, $b^2 - 4 a c > 0$. Hence there are two unique real solutions.

p. 71

[6.1] sufficient

[6.2] necessary and sufficient

[6.1] nessary but not sufficient (suppose $c = 0$)

p. 72

[7] If $a = 2$, $x^2 + (a + 4) x + a^2 + 5 = 0$ is $x^2 + 6 x + 9 = 0$. So, $x = -3$.

If $a = \frac{2}{3}$, $x^2 + \frac{14}{3} x + \frac{49}{9} = 0$ is $x^2 + 6 x + 9 = 0$. So, $x = \frac{-7}{3}$.

[8] $(-2\,a)^2 - (4)\,(2)\,(a^2 - 8) = 0 \Longleftrightarrow a = -4 \vee a = 4$.
If $a = -4$, then $x = -2$. If $a = 4$, then $x = 2$.

P73

[1.1] $\alpha + \beta = -\frac{b}{a} = \frac{1}{1} = 1$

$\alpha\beta = \frac{c}{a} = \frac{1}{1} = 1$

[1.2] $2x^3 + 3x - 4 = 0$

$\alpha + \beta = -\frac{b}{a} = \frac{-3}{2}$

$\alpha\beta = \frac{c}{a} = \frac{-4}{2} = -2$

P74

$2x^2 - 4x + 5 = 0$

[2.1] $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$= \alpha^2 + 2\alpha\beta + \beta^2 - 4\alpha\beta$

$= (\alpha + \beta)^2 - 4\alpha\beta$

$= \left[\frac{4}{2}\right]^2 - 4\left[\frac{5}{2}\right]$

$= 4 - 10$

$= 6$

[2.2] $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha + \beta)(\alpha^2 - 3\alpha\beta + \beta^2 + 2\alpha\beta)$

$= (\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]$

$= \left(\frac{4}{2}\right)\left[\left(\frac{4}{2}\right)^2 - 3\left(\frac{5}{2}\right)\right]$

$= 2\left[4 - \frac{15}{2}\right]$

$= 8 - 15$

$= -7$

p74, ctd

[2,3] $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{2}{\frac{5}{2}} = \dfrac{4}{5}$

[2,4] $\dfrac{\beta}{\alpha-2} + \dfrac{\alpha}{\beta-2}$

$$= \frac{\beta(\beta-2)+\alpha(\alpha-2)}{(\alpha-2)(\beta-2)}$$

$$= \frac{\beta^2-2\beta+\alpha^2-2\alpha}{\alpha\beta-2\beta-2\alpha+4}$$

$$= \frac{\alpha^2+2\alpha\beta+\beta^2 -2(\alpha+\beta)-2\alpha\beta}{\alpha\beta-2(\alpha+\beta)+4}$$

$$= \frac{(\alpha+\beta)^2-2(\alpha+\beta)-2\alpha\beta}{\alpha\beta-2(\alpha+\beta)+4}$$

$$= \frac{2^2-2(2)-2(\frac{5}{2})}{\frac{5}{2}-2(2)+4}$$

$$= \frac{-5}{\frac{5}{2}}$$

$$= -2$$

P75

✓ [3.1] $x^2 - 7x - 98$

$$x = \frac{7 \pm \sqrt{49 + 4(98)}}{2}$$

$$= \frac{7 \pm 7\sqrt{1+8}}{2}$$

$$= \frac{7 \pm 27}{2}$$

$$x^2 - 7x - 98 = (x - 14)(x + 7)$$

✓ [3.2] $6x^2 + 11x - 35$

$$x = \frac{-11 \pm \sqrt{121 + 4(6)35}}{12}$$

$$= \frac{-11 \pm \sqrt{961}}{12}$$

$$= \frac{-11 \pm 31}{12}$$

$$= \left\{ -\frac{7}{2}, \frac{5}{3} \right\}$$

$$\Rightarrow 6x^2 + 11x - 35 = 6\left(x + \frac{7}{2}\right)\left(x - \frac{5}{3}\right)$$

$$= (2x + 7)(3x - 5)$$

p 76

✓ [4.1] $x^2 - x - 1 = 0$

$$\Rightarrow \quad x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$x^2 - x - 1 = \left(x - \tfrac{1}{2} - \tfrac{\sqrt{5}}{2}\right)\left(x - \tfrac{1}{2} + \tfrac{\sqrt{5}}{2}\right)$$

$$= \tfrac{1}{4}\left(2x - 1 - \sqrt{5}\right)\left(2x - 1 + \tfrac{\sqrt{5}}{2}\right)$$

✓ [4.2] $x^2 + 4 = 0$

$$\Rightarrow \quad x^2 = -4$$

$$x = \pm 2i$$

$$x^2 + 4 = (x - 2i)(x + 2i)$$

✓ [4.3] $3x^2 - 2x + 1 = 0$

$$\Rightarrow \quad x = \frac{2 \pm \sqrt{4 - 12}}{6} = \frac{2 \pm \sqrt{-8}}{6} = \frac{2 \pm 2i\sqrt{2}}{6}$$

$$= \frac{1 \pm i\sqrt{2}}{3}$$

$$\Rightarrow \quad 3x^2 - 2x + 1 = \left(x - \tfrac{1}{3} - \tfrac{i\sqrt{2}}{3}\right)\left(x - \tfrac{1}{3} + \tfrac{i\sqrt{2}}{3}\right)$$

$$= \tfrac{1}{9}\left(3x - 1 - i\sqrt{2}\right)\left(3x - 1 + i\sqrt{2}\right)$$

P76

✓ [5.1] $\alpha+\beta = 2+\sqrt{5}+2-\sqrt{5} = 4$

$\alpha\beta = (2+\sqrt{5})(2-\sqrt{5}) = 4-5 = -1$

$\therefore \quad x^2-4x-1=0$

✓ [5.2] $\alpha+\beta = \frac{-5+i}{2}+\frac{-5-i}{2} = \frac{-10}{2} = -5$

$\alpha+\beta = \left(\frac{-5+i}{2}\right)\left(\frac{-5-i}{2}\right) = \frac{25+1}{4} = \frac{26}{4} = \frac{13}{2}$

$\therefore \quad x^2+5x+\frac{13}{2}=0$

P77

✓ [6] $2x^2-x-5=0$, α,β Roots

Find QE whose roots are $\alpha' = 2\alpha-1$, $\beta' = 2\beta-1$

$(\alpha+\beta) = \frac{-b}{a} = -\frac{1}{2}$

$\alpha\beta = \frac{c}{a} = \frac{-5}{2}$

$\alpha'+\beta' = (2\alpha-1)+(2\beta-1) = 2(\alpha+\beta)-2 = 2(\frac{1}{2})-2 = -1$

$\alpha'\beta' = (2\alpha-1)(2\beta-1) = 4\alpha\beta-2\alpha-2\beta+1$

$= 4\alpha\beta-2(\alpha+\beta)+1$

$= 4\left(\frac{-5}{2}\right)-2\left(\frac{1}{2}\right)+1$

$= -10-1+1$

$= -10$

$x^2-(\alpha'+\beta')x+\alpha'\beta'=0$

$\Leftrightarrow \quad x^2-x-10=0$

P 77

[7] α, β roots of $x^2 + ax + b = 0$
Prove QE whose solns are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is $bx^2 + ax + 1 = 0$, where $b \neq 0$.

Proof

$\because$ α, β roots of $x^2 + ax + b = 0$, $b \neq 0$,

and $\alpha' = \frac{1}{\alpha}$, $\beta' = \frac{1}{\beta}$ are roots of $Q(x) = 0$.

Now $\alpha' + \beta' = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

and $\alpha'\beta' = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = \frac{1}{\alpha\beta}$.

Since $\alpha + \beta = -a$ and $\alpha\beta = b$,

$\alpha' + \beta' = \frac{-a}{b}$ and $\alpha'\beta' = \frac{1}{b}$,

thus

$$Q(x) = 0 \iff x^2 - \left(\frac{-a}{b}\right)x + \frac{1}{b} = 0$$

$$\iff bx^2 + ax + 1 = 0$$

$\square$

p78, Exercises

[1] Let $S_1 = \sqrt{a}\sqrt{b} = \sqrt{ab}$
$S_2 = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

[1] S_1, S_2 both true. [2] S_1, S_2 False [3] Both False, [4] Both True

✓ [2.1] $\frac{1}{3}x^2 - x + 2 = 0$

$$x^2 - 3x + 6 = 0$$

$$x = \frac{3 \pm \sqrt{9-24}}{2} = \frac{3 \pm i\sqrt{15}}{2}$$

$$x = \frac{3}{2} + \frac{i\sqrt{15}}{2}, \quad \frac{3}{2} - \frac{i\sqrt{15}}{2}$$

✓ [2.2] $(x+1)^2 + (x+2)^2 = (x-3)^2$ $\quad a^2+b^2=c^2$

$$(x+1)^2 + (x+2)^2 - (x-3)^2 = 0$$

$$\cancel{x^2} + \cancel{2x} + \cancel{1} + \cancel{x^2} + \cancel{4x} + \cancel{4} - [\cancel{x^2} - \cancel{6x} + 9] = 0$$

$$x^2 + 12x - 4 = 0$$

$$\Rightarrow \quad x = \frac{-12 \pm \sqrt{144+16}}{2}$$

$$= \frac{-12 \pm 4\sqrt{10}}{2}$$

$$\therefore \quad x = -6 + 2\sqrt{10}, \ -6 - 2\sqrt{10}$$

[3.1] $16 - 4(-2)(-3) = 16 - 24 < 0$ Two complex roots

[3.2] $-3x^2 + 5x - 2 = 0$

$D = 25 - 4(-3)(-2) = 25 - 24 \Rightarrow$ Two Real roots

[3.3] $3x^2 - mx - 1 = 0$

$D = m^2 - 4(3)(-1) = m^2 + 12$

Now $m^2 + 12 = 0 \Rightarrow m = \pm 2i\sqrt{3} \Rightarrow \sim\exists\, m \in \mathbb{R} \ni D = 0$

$m^2 + 12 < 0 \Rightarrow m^2 < -12 \Rightarrow \sim\exists\, m \in \mathbb{R} \ni D < 0$

$m^2 + 12 > 0 \Rightarrow m^2 > -12 \Rightarrow \forall m \in \mathbb{R} \ni D > 0$

$\therefore$ Two real roots for all $m \in \mathbb{R}$.

[3.4] $x^2 - 4ax + 5a^2 = 0$

$D = 16a^2 - 4(1)(5a^2)$

$= 16a^2 - 20a^2$

$= -4a^2$

and $-4a^2 \leq 0, \ \forall a \in \mathbb{R} \because a^2 \geq 0$

$\therefore$ one soln multiplicity 2 when $a = 0$

two complex solns for $a < 0$ or $a > 0$,

i.e. when $|a| \neq 0$.

$\longrightarrow$

P 78, ctd

[4.1] sufficient

[4.2] if $q<0$ then $p^2-4q>0$. Sufficient.

Necessary?

Does $p^2-4q>0 \Rightarrow q<0$?

If $p^2-4q>0$. Then $p^2>4q$, which is true when $p=3$ and $q=1$ ($q>0$).

So not necessary. I.e. $x^2+3x+1=0$ has 2 real soln, but $q \nless 0$

✓ [5.1] α, β roots of $x^2+ax+3=0$

get $\underline{a}$ s.t. $\alpha^2+\beta^2=3$.

$$\alpha+\beta = -a$$
$$\alpha\beta = 3$$

$$\alpha^2+\beta^2 = \alpha^2+2\alpha\beta+\beta^2-2\alpha\beta$$
$$= (\alpha+\beta)^2-2\alpha\beta$$
$$= (-a)^2-2(3)$$
$$\alpha^2+\beta^2 = a^2-6$$

Now, if $\alpha^2+\beta^2=3$ then

$$a^2-6=3 \Rightarrow a^2=9$$
$$\Rightarrow a=\pm 3$$

$\therefore\ a=\pm 3$

p 78, ctd

[6] Let x = width
$y = x+1$ = Length
$A = xy = 42$

$$x(x+1) = 42$$
$$x(x+1) - 42 = 0$$
$$x^2 + x - 42 = 0$$
$$(x-7)(x+6) = 0$$
$$x = 7 \text{ or } x = -6$$

$\therefore$ width = 7 cm, Len = 8 cm

✓ [7] Given α, β solutions of $2x^2 - 3x + 2 = 0$.
Get Q.E. $Q(x) = 0$ s.t. $\alpha' = 2\alpha+1$, $\beta' = 2\beta+1$.

Soln

$$\alpha + \beta = \frac{3}{2}, \quad \alpha\beta = 1$$

$$Q(x) = x^2 - (\alpha'+\beta')x + \alpha'\beta'$$

$$\alpha' + \beta' = 2\alpha + 1 + 2\beta + 1 = 2\alpha + 2\beta + 2 = 2\left(\frac{3}{2}\right) + 2 = 5$$

$$\begin{aligned} \alpha'\beta' = (2\alpha+1)(2\beta+1) &= 4\alpha\beta + 2\alpha + 2\beta + 1 \\ &= 2(\alpha+\beta) + 4\alpha\beta + 1 \\ &= 2\left(\frac{3}{2}\right) + 4(1) + 1 \\ &= 3 + 4 + 1 \\ &= 8 \end{aligned}$$

So

$$Q(x) = x^2 - 5x + 8 = 0$$

P 78, ctd

[8] Get k, s.t. $\frac{\alpha}{\beta} = \frac{3}{2}$;

α, β solutions of

$$2x^2 - kx + k + 2 = 0$$

<u>SOLN</u>

$$\alpha + \beta = \frac{k}{2}, \quad \alpha\beta = \frac{k+2}{2}$$

$$\alpha = \frac{3}{2}\beta$$

$$\Rightarrow \begin{bmatrix} \frac{3}{2}\beta + \beta = \frac{k}{2} \\ \frac{3}{2}\beta^2 = \frac{k+2}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 5\beta = k \\ 3\beta^2 = k+2 \end{bmatrix} \Rightarrow \beta = \frac{k}{5}$$

$$\Rightarrow 3\left(\frac{k}{5}\right)^2 = k+2$$

$$\Rightarrow \frac{3k^2}{25} = k+2$$

$$\Rightarrow 3k^2 = 25k + 50$$

$$3k^2 - 25k - 50 = 0$$

$$\Rightarrow k = 10 \quad \text{OR} \quad k = \frac{-5}{3}$$

<u>CHECK</u>

$k = 10$

$2x^2 - 10x + 12 = 0$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2, 3$

$\frac{\alpha}{\beta} = \frac{3}{2}$ ✓

$k = \frac{-5}{3}$

$2x^2 + \frac{5}{3}x - \frac{5}{3} + 2 = 0$

$6x^2 + 5x - 5 + 6 = 0$

$6x^2 + 5x + 1 = 0$

$(x - \frac{1}{2})(x - \frac{1}{3}) = 0$

$\frac{\alpha}{\beta} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2}$ ✓

$$\boxed{\therefore k = 10 \text{ OR } k = \frac{-5}{3}}$$

Page 80

[1.1] $x = 2,\ y = -1,\ z = 3$

[1.2] $x = -2,\ y = 5,\ z = 3$

[1.3] $x = \frac{22}{9},\ y = \frac{2}{3},\ z = \frac{8}{9}$

P 81

[1.1] $\begin{bmatrix} x - 1 = y \\ x^2 + y^2 = 25 \end{bmatrix} \Rightarrow x^2 + (x-1)^2 = 25$

$\Rightarrow x^2 + x^2 - 2x + 1 = 25$

$\Rightarrow 2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$\boxed{x = 4, -3}$

$4 - 1 = 3$

$-3 - 1 = -4$

$\therefore\ (x, y) = (4, 3), (-3, -4)$

[1.2] $\begin{bmatrix} x + 3y = 5 \\ x + y^2 = 3 \end{bmatrix} \Rightarrow (3 - y^2) + 3y = 5$

$-y^2 + 3y - 2 = 0$

$y^2 - 3y + 2 = 0$

$(y-1)(y-2) = 0$

$\boxed{y = 1, 2}$

$x + 3 = 5 \Rightarrow x = 2$

$x + 6 = 5 \Rightarrow x = -1$

$\therefore\ (x, y) = (1, 2), (2, -1)$

$\Rightarrow$

P81, ctd

[1.3] $\begin{bmatrix} y = \sqrt{3}\,x \\ x^2+y^2=48 \end{bmatrix} \Rightarrow x^2+3x^2=48$

$4x^2=48$

$x^2=12$

$\boxed{x = 2\sqrt{3},\ -2\sqrt{3}}$

which EQ to sub into ?

$12+y^2=48$

$y^2=36$

$\boxed{y=\pm 6}$

$x=2\sqrt{3} \Rightarrow y=\sqrt{3}(2\sqrt{3}) = 2\cdot 3 = 6$

$x=-2\sqrt{3} \Rightarrow y=\sqrt{3}(-2\sqrt{3}) = -6$

Into EQ1 $\Rightarrow$ $(x,y) = (2\sqrt{3},6), (-2\sqrt{3},-6)$

$x=2\sqrt{3} \Rightarrow 12+y^2=48 \Rightarrow y=\pm 6$

$x=(-2\sqrt{3}) \Rightarrow y=\pm 6$

Into EQ 2. But cannot tell how the ordered pairs are formed.

* EQ2 Does not distinguish $2\sqrt{3}$ AND $-2\sqrt{3}$. Both go to ~~E~~. 6.

$\therefore\ (x,y) = (2\sqrt{3},6), (-2\sqrt{3},-6)$

p 81, ctd

✓[2]

$$\begin{bmatrix} x^2 + y^2 = (13)^2 \\ x + y \quad = 17 \end{bmatrix} \Rightarrow (17-y)^2 + y^2 = 169$$

$$289 - 34y + y^2 + y^2 = 169$$

$$2y^2 - 34y + 120 = 0$$

$$y^2 - 17y + 60 = 0$$

$$(y-12)(y-5) = 0$$

$$\boxed{y = 12, 5}$$

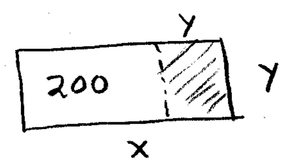
$$x + 12 = 17 \Rightarrow x = 5$$

$$x + 5 = 17 \Rightarrow x = 12$$

$\therefore (x, y) = (5, 12)$ { (5, 12) and (12, 5) are same Δ.

✓

[3]



$x + y = 100$
$x > y$

$$\begin{bmatrix} xy - y^2 = 200 \\ x + y \quad = 50 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y = 50 - x \\ xy - y^2 = 200 \end{bmatrix}$$

$$\Rightarrow (50-x)x - (50-x)^2 = 200$$

$$50x - x^2 - [2500 - 100x + x^2] - 200 = 0$$

$$5\cancel{0}x - \cancel{x^2} - 2500 + 10\cancel{0}x - \cancel{x^2} - 200 = 0$$

$$-2x^2 + 150x - 2700 = 0$$

$$x^2 - 75x + 1350 = 0$$

$$(x-45)(x-30) = 0$$

$$\boxed{x = 45, 30}$$

$x = 45 \Rightarrow 45 + y = 50 \Rightarrow y = 5$

$x = 30 \Rightarrow 30 + y = 50 \Rightarrow y = 20$

$\therefore (x, y) = (45, 5), (30, 20)$

p 81, ctd

careful $\qquad x^2 - px + q = 0, \quad p = \alpha + \beta, \quad q = \alpha\beta$

[4,1] $\begin{bmatrix} x + y = 11 \\ xy = 30 \end{bmatrix} \Rightarrow t^2 - 11x + 30 = 0$

$\Rightarrow (t-5)(t+6) = 0$

$\Rightarrow t = 5, 6$

$\therefore (x, y) = (5, 6), (6, 5)$

[4,2] $\begin{bmatrix} x + y = -\frac{20}{3} \\ xy = 4 \end{bmatrix} \Rightarrow t^2 + \frac{20}{3} t + 4 = 0$

$3t^2 + 20t + 12 = 0$

$\Rightarrow t = -\frac{2}{3}, -6$

$\therefore (x, y) = (-\frac{2}{3}, -6), (-6, -\frac{2}{3})$

P 82

[1] $\omega = \dfrac{-1+\sqrt{3}\,i}{2}$

$$\omega^2 = \frac{1-2\sqrt{3}\,i-3}{4} = \frac{-2-2\sqrt{3}\,i}{4} = \frac{-1-\sqrt{3}\,i}{2}$$

✓ [2.1]

$$\begin{array}{r} x^2+2x+4 \\ x-2 \,\overline{)\, x^3-2^3} \\ \underline{x^3-2x^2} \\ 2x^2 \\ \underline{2x^2-4x} \\ 4x \\ 4x-8 \end{array}$$

$x^3 = 8$

$x^3 - 8 = 0$

$x^3 - 2^3 = 0$

$\Rightarrow (x-2)(x^2+2x+4) = 0$

Then

$x = 2$ OR $x^2+2x+4=0$

$$x = \frac{-1 \pm \sqrt{1-4}}{1}$$

$$= -1+i\sqrt{3},\ -1-i\sqrt{3}$$

$$\boxed{\therefore\ x^3 = 8 \Rightarrow x = 2,\ -1+i\sqrt{3},\ -1-i\sqrt{3}}$$

[2,2] $x^3 = -1 \Rightarrow x^3+1=0 \Rightarrow (x-1)(x^2+x+1)=0$

$$\Rightarrow x=1 \text{ or } x = \frac{-1\pm\sqrt{1-4}}{2}$$

$$\boxed{\Rightarrow x = 1,\ -\frac{1}{2}+\frac{i\sqrt{3}}{2},\ -\frac{1}{2}-\frac{i\sqrt{3}}{2}}$$

p 83, ctd

[2.3]

$$x^4 = -8x \Rightarrow x^4 + 8x = 0$$

$$\Rightarrow \quad x(x^3 + 8) = 0$$

$$x(x+2)(x^2 - 2x + 4) = 0$$

$$x^2 - 2x + 4 = 0 \Rightarrow \quad x = \frac{2 \pm \sqrt{4-16}}{2}$$

$$= \frac{2 \pm 2i\sqrt{3}}{2}$$

$$= 1 \pm i\sqrt{3}$$

$$\therefore \quad x = 0, -2, 1 + i\sqrt{3}, 1 - i\sqrt{3}$$

[3.1]

$$x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x(x-3)(x+3) = 0$$

$$\therefore \quad x = 0, 3, -3$$

[3.2]

$$x^4 - 1 = 0$$

$$(x^2+1)(x^2-1) = 0$$

$$(x^2+1)(x-1)(x+1) = 0$$

$$\therefore \quad x = 1, -1, i, -i$$

4th roots of unity.

P 83, ctd

✓ [3.3]

$$x^3 + 5x^2 - 8x - 40 = 0$$

$$x^2(x+5) - 8(x+5) = 0$$

$$(x+5)(x^2 - 8) = 0$$

$$\therefore\ x = -5,\ 2\sqrt{2},\ -2\sqrt{2}$$

✓ [3.4]

$$x^4 - 3x^2 + 2 = 0$$

$$y = x^2$$

$$y^2 - 3y + 2 = 0$$

$$(y-1)(y-2) = 0$$

$$(x^2-1)(x^2-2) = 0$$

$$(x-1)(x+1)(x-\sqrt{2})(x+\sqrt{2}) = 0$$

$$\therefore\ x = 1,\ -1,\ -\sqrt{2},\ \sqrt{2}$$

---

[2] a) $(x^3+2x-12)\div(x+1) = (-1)^3-2-12 = -15$

b) $\div(x-1) = (1)^3+2-12 = -9$

c) $\div(x-2) = 2^3+4-12 = 8+4-12 = 0$

[3] If R remainder of $f(x)$ div by $ax+b$,
then $R = f(\frac{-b}{a})$.

<u>Proof</u>

$f(x) = (ax+b)g(x) + R$, since R remainder of $f(x)$ div by $ax+b$. Then,
$f(x) = a(x+\frac{b}{a})g(x) + R$. Let $x = -\frac{b}{a}$, so
$f(-\frac{b}{a}) = a(-\frac{b}{a}+\frac{b}{a})g(x) + R = R$.
$\therefore$ $f(x)$ divided by $ax+b$, then $f(-\frac{b}{a}) = R$

$\square$

[4] $(4x^3-2x^2-9)\div A$

a) $A = 2x-1$, $R = f(\frac{1}{2}) = \frac{1}{2}-\frac{1}{2}-9 = -9$ $\therefore R = -9$

b) $A = 2x+1$, $R = f(-\frac{1}{2}) = -\frac{1}{2}-\frac{1}{2}-9 = -10$ $\therefore R = -10$

c) $A = 2x+3$, $R = f(-\frac{3}{2}) = -\frac{27}{2}-\frac{9}{2}-9 = -27$ $\therefore R = -27$

P86

[5] Is $X-3$ or $X+3$ a factor of

[5.1] x^3-2x^2-5x+6?

$P(3) = 27-18-15+6 = 0 \Rightarrow$ YES $(X-3)$ FACTOR

$P(-3) = -27-18+15+6 = -24 \neq 0 \Rightarrow$ NO $(X+3)$ NOT Factor

[5.2] $x^4+x^3-8x^2-9x-9$

$P(3) = 81+27-72-27-9 = 0 \Rightarrow$ YES $(X-3)$ Factor

$P(-3) = 81-27-72+27-9 = 0 \Rightarrow$ YES $(x+3)$ Factor

✓[6] $P(x) = x^4-2x+\alpha$. Get α s.t. $x+2$ divisor.

SOLN

$P(-2) = 16+4+\alpha = 0$

$\Rightarrow \alpha = -20$

$\boxed{\therefore \alpha = -20}$

$\longrightarrow$

[7] $f(x) \div (x-4) \Rightarrow R = 0$
$f(x) \div (x+3) \Rightarrow R = 14$
Get R when $f(x) \div (x^2 - x - 12)$.

Soln

Since divisor is quadratic, degree $R \leq 1$.
Then,

$$f(x) = Q(x)(x-4)(x+3) + ax + b$$
$$f(4) = Q(x)(x-4)(x+3) + ax+b \Rightarrow 4a+b=0$$
$$f(-3) = Q(x)(x-4)(x+3) + ax+b \Rightarrow -3a+b=14$$

$$\begin{bmatrix} 4a+b=0 \\ -3a+b=14 \end{bmatrix} \Rightarrow 7a = -14 \Rightarrow \begin{bmatrix} a=-2 \\ b=8 \end{bmatrix}$$

$$\therefore \quad f(x) \div (x^2 - x - 12) \Rightarrow R = -2x+8$$

[8] If $f(x)$ divisible by $x-3$ and by $x-5$,
then $f(x)$ divisible by $(x-3)(x-5)$.

Proof

$\oint$ $f(x)$ div by $x-3$ and by $x-5$,
Then $f(3) \Rightarrow R=0$ and $f(5) \Rightarrow R=0$.
Since Degree $R \leq 1$ when $f(x)$ div by $[(x-3)(x-5)]$

$$f(x) = Q(x)(x-3)(x-5) + ax+b$$

$$\text{But} \begin{bmatrix} f(3) \Rightarrow 3a+b=0 \\ f(5) \Rightarrow 5a+b=0 \end{bmatrix} \Rightarrow 2a=0 \Rightarrow \begin{bmatrix} a=0 \\ b=0 \end{bmatrix},$$

so $f(x) = Q(x)(x-3)(x-5) + 0$

which means $[(x-3)(x-5)]$ divides $f(x)$.

$\square$

J10
Answers
PPS 89– 90

[1.1] $x^3 - 4x^2 - 3x + 18 = 0$

$f(3) = 0$

$$\begin{array}{rrrr|l} 1 & -4 & -3 & +18 & \underline{3} \\ & 3 & -3 & -18 & \\ \hline 1 & -1 & -6 & 0 & \end{array}$$

So $x^3 - 4x^2 - 3x + 18 = 0$

$\Leftrightarrow (x-3)(x^2 - x - 6) = 0$

$\Leftrightarrow (x-3(x-3)(x-2) = 0$

$\therefore x = 2, 3$

[1.2] $2x^3 - 7x^2 + 7x - 2 = 0$

$f(1) = 0$

$$\begin{array}{rrrr|l} 2 & -7 & 7 & -2 & \underline{1} \\ & 2 & -5 & 2 & \\ \hline 2 & -5 & 2 & 0 & \end{array}$$

So $2x^3 - 7x^2 + 7x - 2 = 0$

$\Leftrightarrow (x-1)(2x^2 - 5x + 2) = 0$

$$x = \frac{5 \pm \sqrt{25-16}}{4}$$

$$= \frac{5 \pm 3}{4}$$

$$= 2, \tfrac{1}{2}$$

$\therefore x = \frac{1}{2}, 1, 2$

P 89, ctd

[1.3]

$6x^3+5x^2-12x+4=0$

$f(-2)=0$

$$\begin{array}{cccc|l} 6 & 5 & -12 & 4 & \underline{\lfloor -2} \\ & -12 & 14 & -4 & \\ \hline 6 & -7 & 2 & 0 & \end{array}$$

$6x^3+5x^2-12x+4=0$

$\Rightarrow \quad (x+2)(6x^2-7x+2)=0$

$$x = \frac{7 \pm \sqrt{49-48}}{12}$$

$$= \frac{7 \pm 1}{12}$$

$$= \frac{2}{3}, \frac{1}{2}$$

$\therefore \quad x = -2, \frac{1}{2}, \frac{2}{3}$

[1.4] $\quad x^4+x^3-x^2+x-2=0$

$f(1)=0$

$$\begin{array}{ccccc|l} 1 & 1 & -1 & 1 & -2 & \underline{\lfloor 1} \\ & 1 & 2 & 1 & 2 & \\ \hline 1 & 2 & 1 & 2 & 0 & \end{array}$$

$g(x) = x^3+2x^2+x+2$

$g(-2)=0$

$$\begin{array}{cccc|l} 1 & 2 & 1 & 2 & \underline{\lfloor -2} \\ & -2 & 0 & -2 & \\ \hline 1 & 0 & 1 & 0 & \end{array}$$

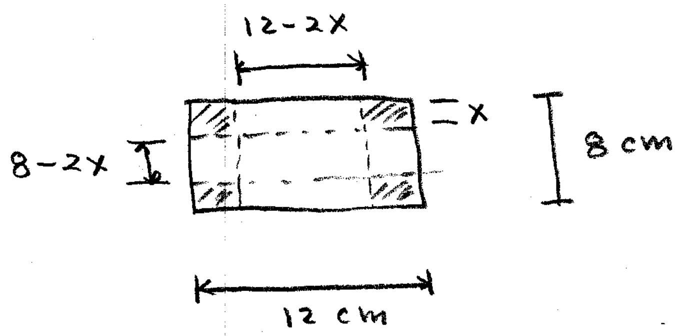
$x^4+x^3-x^2+x-2=0$

$\Leftrightarrow \quad (x-1)(x+2)(x^2+1)=0$

$\boxed{\therefore \quad x = -2, 1, -i, i}$

P 90

[2]



$V = x(12-2x)(8-2x)$

$V = 36\ cm^3$

$\Rightarrow \quad (x)(12-2x)(8-2x) = 36$

$\Leftrightarrow \quad 4x^3 - 40x^2 + 96x - 36 = 0$

$\Leftrightarrow \quad x^3 - 10x^2 + 24x - 9 = 0$

$f(3) = 0$

1	-10	24	-9	3
	3	-21	9	
1	-7	3	0	

$x^2 - 7x + 3 = 0$

$x = \frac{7 \pm \sqrt{49-12}}{2} = \frac{7 \pm \sqrt{37}}{2}$

$\Leftrightarrow \quad x^3 - 10x^2 + 24x - 9 = 0$

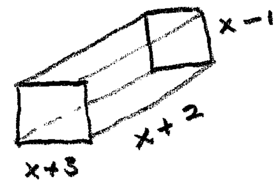
$\therefore \quad x = 3, \frac{7-\sqrt{37}}{2}, \frac{7+\sqrt{37}}{2}$

p 90, ctd

[3]

$V_0 = x^3$ } cube

$h = x+1$
$w = x-3$
$l = x-2$



$V = 60\ cm^3$

$V = hwl$

$(x-1)(x+2)(x+3) = 60$

$\Leftrightarrow \quad x^3 + 4x^2 + x - 6 = 60$

$\Leftrightarrow \quad x^3 + 4x^2 + x - 66 = 0$

$f(3) = 0$

$$\begin{array}{rrrr|l} 1 & 4 & 1 & -66 & \underline{3} \\ & 3 & 21 & 66 & \\ \hline 1 & 7 & 22 & 0 & \end{array}$$

$x^2 + 7x + 22 = 0$

$x = \dfrac{-7 \pm \sqrt{49-88}}{2}$, These values of x are complex and so cannot be lengths of sides of box.

$x = 3$, so

∴ original cube of side 3 cm.

[08-03-27-J10]

Answers to page 91

[1.1] $(x, y, z) = (3, 4, 1)$

[1.2] $(x, y) = (4, -3)$

[2.1] $x = 2, \frac{1-\sqrt{17}}{4}, \frac{1+\sqrt{17}}{4}$

[2.2] $x = \frac{-3}{5}, 1, 2$

[2.3] $x = -\frac{1}{2}, \frac{1}{2}, \frac{2}{3}$

[2.4] $x = -1, 1, -4-\sqrt{6}, -4+\sqrt{6}$

Complete solutions to problems #3-8 on following pages

[3] Get k s.t. only one pair of solutions to system of equations

$$\begin{bmatrix} 2x + y = k \\ x^2 + y^2 = 5 \end{bmatrix}.$$

(note: "one pair of solutions" means one ordered pair (x, y) such that values of x, y statisfy the System of equations given.)

$$\begin{bmatrix} 2x+y=k \\ x^2+y^2=5 \end{bmatrix} \Longrightarrow 5x^2 - 4kx + k^2 - 5 = 0.$$

one pair of values (x,y) implies a single value of x.

So $D = 0$.

$$D = 16k^2 - 20k^2 + 100 = 0$$

$$-4k^2 + 100 = 0$$

$$k^2 - 25 = 0$$

$$k = 5 \text{ OR } k = -5$$

Then $(k=5)$

$$\begin{bmatrix} 2x + y = 5 \\ x^2 + y^2 = 5 \end{bmatrix} \Rightarrow (x-2)^2 = 0 \Rightarrow x = 2, y = 1, \text{ so } (x,y) = (2,1)$$

alteratively $(k = -5)$

$$\begin{bmatrix} 2x + y = -5 \\ x^2 + y^2 = 5 \end{bmatrix} \quad x^2 + 4x + 4 = 0 \Rightarrow x = -2, y = -1$$

$\therefore$ when $k = 5, -5$, the system of equations has one pair of Solutions

[4] $x^2 + 2x + p$, $x^2 + (p+8)x + q$.

Suppose $x-2$ divisor of both expressions. Then,

$f(x) = x^2 + 2x + p$

$f(2) = 0$, since $x-2$ factor.

$$2^2 + 2\cdot 2 + p = 0$$

$$\boxed{p = -8}$$

$g(x) = x^2 + (-8+8)x + q$

$g(2) = 0$

$$2^2 + q = 0$$

$$\boxed{q = -4}$$

So $f(x) = x^2 + 2x - 8$

$g(x) = x^2 - 4$

LCM $[(x^2-4), (x^2+2x-8)]$

$$x^2 - 4 = (x-2)(x+2)$$

$$x^2 + 2x - 8 = (x-2)(x+4)$$

$$\text{Lcm}[f(x), g(x)] = (x-2)(x+2)(x+4)$$

$$= x^3 + 4x^2 - 4x - 16$$

$\therefore$ $x^3 + 4x^2 - 4x - 16$ is LCM $[(x^2-4), (x^2+2x-8)]$

P91, ctd

[5]

$f(x) = x^3 + 2ax^2 - (a+2b)x - 3b$

$f(x)$ divisible by $x+1$

$\Rightarrow f(-1) = -1 + 2a + a + 2b - 3b = 0$

$f(-1) = \boxed{3a - b = 1}$

$f(x)$ divisible by $x-2$

$\Rightarrow f(2) = 8 + 8a - 2(a+2b) - 3b = 0$

$\Rightarrow 8 + 8a - 2a - 4b - 3b = 0$

$\Rightarrow \boxed{6a - 7b = -8}$

solve for a, b

$$\begin{bmatrix} 3a - b = 1 \\ 6a - 7b = -8 \end{bmatrix} \Rightarrow \begin{bmatrix} 6a - 2b = 2 \\ 6a - 7b = -8 \end{bmatrix} \Rightarrow 5b = 10 \Rightarrow \boxed{b=2} \quad \therefore \boxed{a=1}$$

For $a=1, b=2$

$f(x) = x^3 + 2x^2 + 3x - 6$

$f(x) = 0$

$x^3 + 2x^2 + 3x - 6 = 0$

$\therefore x = 1, \frac{3+i\sqrt{15}}{2}, \frac{3-i\sqrt{15}}{2}$

[6]

$f(x) \doteq (x+4)\,g(x)$

$g(x) = (x-3)\,h(x) + 5.$ Get R when $f(x) \div (x-3)$

$\Rightarrow$

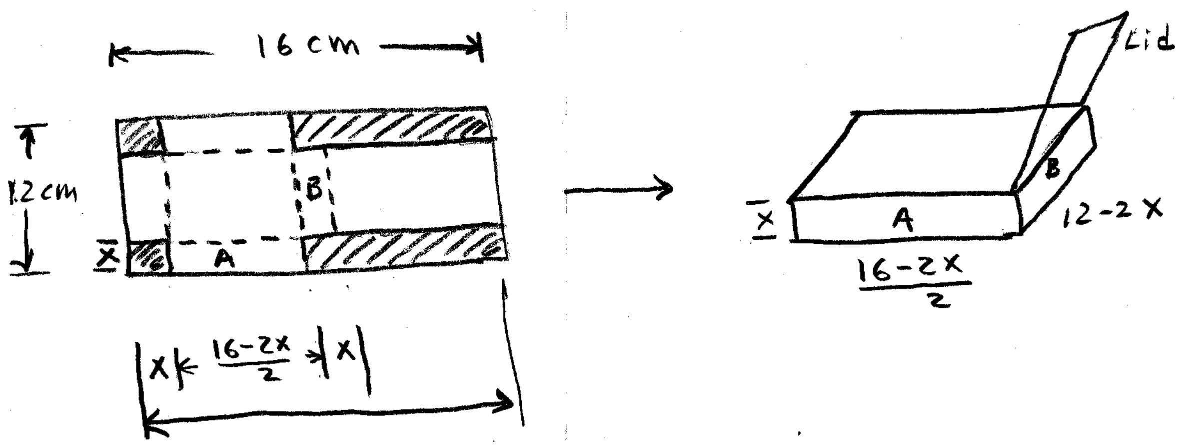
$f(x) = (x+4)\left[(x-3)\,h(x)+5\right]$

$f(3) = 7\left[0+5\right]$

$= 35$

$\therefore$ when $f(x)$ divided by $x-3$, Remainder is 35.

[7]



$V(x) = x\left(\frac{16-2x}{2}\right)(12-2x)$ cm^3

for $V(x) = 96$ cm^3

$x\left[\frac{16-2x}{2}\right](12-2x) = 96$

$x(8-x)(6-x) = 48$

$x^3 - 14x^2 + 48x - 48 = 0$

$x = 2,\ 6-2\sqrt{3},\ 6+2\sqrt{3}$

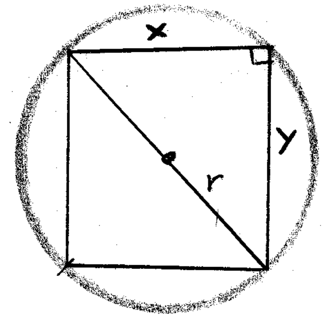
But $12-(6+2\sqrt{3}) < 0$, SO NO BOX

$\therefore$ $x = 2,\ 6-2\sqrt{3}$

GIVEN

#8

$$\begin{bmatrix} x + y = 21 \\ x^2 + y^2 = 900 \end{bmatrix}$$



$x + y = 21$ cm
$r = 15$ cm
$2r = 30$ cm

$$\begin{bmatrix} y = 21 - x \\ x^2 + y^2 = 900 \end{bmatrix}$$

$$x^2 + (21-x)^2 = 900$$

$$x^2 + 441 - 42x + x^2 = 900$$

$$2x^2 - 42x - 459 = 0$$

so

$$x = \frac{-21 \pm \sqrt{21^2 + 918}}{2}$$

$$= \frac{-21 \pm \sqrt{1359}}{2}$$

$$x = \frac{-21 + 3\sqrt{151}}{2}, \quad \frac{-21 - 3\sqrt{151}}{2}$$

$\sqrt{151} > \sqrt{144} = 12$, so 2nd root is neg and 1st root is positive. Length cannot be negative.

$$x = \frac{-21 + 3\sqrt{151}}{2}$$

$$y = \frac{21 + 21 - 3\sqrt{151}}{2} = 21 - \frac{3\sqrt{151}}{2}$$

$$\boxed{\therefore \quad x = \frac{-21 + 3\sqrt{151}}{2}, \quad y = 21 - \frac{3\sqrt{151}}{2}}$$

these are approx

$$\left.\begin{array}{l} x \approx 8 \\ y \approx 13 \end{array}\right\} \begin{array}{l} 8 + 13 = 21 \\ \sqrt{64+169} \approx 15 \end{array}$$