

Solutions to J10 page 102 exercises

[1.1] $x \in (-\infty, 3) \cup (4, \infty +)$

[1.2] $x \in (2 - \sqrt{11}, 2 + \sqrt{11})$

[1.3] $x \in (-\infty, 4 - \sqrt{13}] \cup [4 + \sqrt{13}, \infty +)$

[1.4] $x = 6$

[1.5] $x \in \mathbb{R}$ or you can write $x \in (-\infty, \infty +)$

[1.6] $x \in \emptyset$

[1.7] $x \in (\frac{1}{2}, 3)$

[1.8] $x \in \emptyset$

[1.9] $x \in (-\infty, -4 - \sqrt{17}) \cup (-4 + \sqrt{17}, \infty +)$

[1.10] $x \in [-10, -3]$

Completely worked solutions to the rest of the questions on page 102 follow.

[2] $P = 40 \text{ cm} = 2x + 2y$

$64 \text{ cm}^2 \leq A = xy \leq 91 \text{ cm}^2$.

Get width.

$$\begin{cases} x + y = 20 \\ xy \leq 91 \end{cases} \Rightarrow \begin{cases} y = 20 - x \\ xy \leq 91 \end{cases}$$

$$\begin{aligned} \Rightarrow x(20 - x) &\leq 91 \\ -x^2 + 20x - 91 &\leq 0 \\ x^2 - 20x + 91 &\geq 0 \end{aligned}$$

$\Rightarrow x \leq 7$ or $x \geq 13$

But $x > 0$, else no rectangle. x can be 7, since $40 - 14 = 26$ is a possible rectangle $7 \times 13 \text{ cm}$

\therefore so width must be no greater than 7 cm.

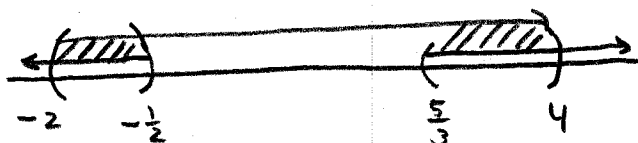
[3] $M = \{x \mid 6x^2 - 7x - 5 > 0\}$

$N = \{x \mid x^2 - 2x - 8 < 0\}$

Get all integers in $M \cap N$

M
 $6x^2 - 7x - 5 > 0$
 $x \in (-\infty, -\frac{1}{2}) \cup (\frac{5}{3}, \infty)$

AND
 N
 $x^2 - 2x - 8 < 0$
 $(x - 4)(x + 2) < 0$
 $x \in (-2, 4)$



$\therefore x = \{-1, 2, 3\}$

P102, c+d

[4] $x^2 + 2(m-4)x + (3m-2) = 0$. Get $m \in \mathbb{R} \Rightarrow$ SOLNS \mathbb{R} .

$$D \geq 0$$

$$(m-4)^2 - (3m-2) \geq 0$$

$$m^2 - 8m + 16 - 3m + 2 \geq 0$$

$$m^2 - 11m + 18 \geq 0$$

$$(m-9)(m-2) \geq 0$$

$$\therefore m \in (-\infty, 2) \cup (9, \infty)$$

[5] Solve $x^2 - (a+2)x + 2a < 0$ for $a > 2$

$$x = \frac{(a+2) \pm \sqrt{(a+2)^2 - 8a}}{2}$$

$$= \frac{a+2 \pm \sqrt{a^2 + 4a + 4 - 8a}}{2}$$

$$= \frac{a+2 \pm \sqrt{a^2 - 4a + 4}}{2}$$

$$= \frac{a+2 \pm (a-2)}{2}$$

$$= \frac{a+2+a-2}{2}, \frac{a+2-a+2}{2}$$

$$= a, 2$$

$$x \in (-\infty, 2) \cup (a, \infty)$$

$$\therefore x \in (-\infty, 2) \cup (2, \infty) \text{ or } x \in \mathbb{R} - \{2\}$$

In other words when $a > 0$,
the soln of the inequality
is all real numbers except 2

P 102, Q6

[6] get p, q s.t. $2x^2 + px + q < 0$ has soln $x \in (\frac{1}{2}, 4)$

Soln

$\frac{1}{2}$ and 4 must be the roots of $2x^2 + px + q = 0$.

$$\begin{aligned} \text{So, } 2x^2 + px + q &= 2(x - \frac{1}{2})(x - 4) \\ &= (2x - 1)(x - 4) \\ &= 2x^2 - 9x + 4 \end{aligned}$$

$$\therefore p = -9 \text{ and } q = 4$$