Theorems

For all $a, b, c \in \mathbb{R}$,

- **Thm G1**

  $$a = b \Longrightarrow a \cdot c = b \cdot c \quad \text{and} \quad a = b \Longrightarrow a + c = b + c$$

- **Thm G2 (Cancellation Theorem)**

  $$a \cdot c = b \cdot c \Longrightarrow a = b, \; c \neq 0 \quad \text{and} \quad a + c = b + c \Longrightarrow a = b$$

- **Thm F1**

  $$0 \cdot a = 0$$

- **Thm F2**

  if $a \cdot b = 0$, then at least one member of $\{a, b\}$ must be zero.

- **Thm F3**

  $$a \cdot (-b) = -(a \cdot b)$$

- **Thm F4**

  $$(-a)(-b) = a\, b$$

- **Thm F5**

  $$-(-a) = a$$

- **Thm F6**

  $$-1 \cdot a = -a$$

Theorems Concerning Fractions

For all $a, b, c, d \in \mathbb{R}$,

◆ **Theorem R1**

$$\frac{1}{a\,b} = \frac{1}{a} \cdot \frac{1}{b}, \text{ where } a\,b \neq 0$$

◆ **Theorem R2**

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a\,c}{b\,d}, \text{ where } b\,d \neq 0$$

◆ **Theorem R3**

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}, \text{ where } b \neq 0$$

Propositions

For all $a, b \in \mathbb{R}$,

◆ **P1**

$$a = \frac{a}{1}$$

◆ **P2**

$$\frac{a}{a} = 1, \text{ where } a \neq 0$$

◆ P3

$$\frac{1}{\frac{a}{b}} = \frac{b}{a}, \qquad \text{where } a\,b \neq 0$$

■ **Note that theorem F1 implies that division by 0 is undefined. Since $0 \cdot a = 0$ for all $a \in \mathbb{R}$, there exists no number a such that $0 \cdot a = 1$. That is, 0 has no multiplicative inverse.**