[12-01-22-T7]

Functions

The idea of a function has evolved over a period of time in mathematics. Eventually, you will learn several of the ways mathematicians have thought of functions. For now, we will think of a function as a rule that pairs an item with exactly one other item. For us, this will nearly always be an equation that pairs a number with exactly one other number.

EXAMPLE 1. Suppose a train travels at a constant speed of 60 miles per hour. We can express the distance, represented by the letter *s* as a function of time, represented by the letter *t*, in the following way: $s = 60\ \frac{\text{mi}}{h}\ t$, where the unit for *s* is the mile and the unit for *t* is the hour.

EXAMPLE 2. Suppose a tank is filled at the rate of 2 liters per minute. We can express the volume, represented by the letter *V* as a function of time, represented by the letter *t*, in the following way: $V = 2\ \frac{L}{\text{min}}\ t$, where the unit for *s* is the liter and the unit for *t* is the minute.

EXAMPLE 3. Suppose a tank containing 300 liters of water is drained at the rate of 2 liters per minute. We can express the volume *V* liters of water in the tank as a function of time *t* minutes as $V = 300\,L - 2\ \frac{L}{\text{min}}\ t$.

If you compare the language of Example 3 to that of Example 2, you will see that we found a less tedious way to say which letters and units go with each quantity.

EXAMPLE 4. Suppose a bakery sells donuts for 60¢ each. Then the gross profit *$P* from selling *n* number of donuts is $P = 0.60\ \frac{\$}{\text{donut}} \times n$.

EXAMPLE 5. Suppose the cost of producing a donut is 40¢ and the bakery sells donuts for 60¢ each. Then the net profit *P* from selling *n* number of donuts is $P = 0.20 $\frac{\$}{\text{donut}} \times n$.

EXAMPLE 6. Two trains run toward each other on parallel tracks (so they do not collide). If the trains are 1 mile apart at noon and one train travels at $110\ \frac{f}{s}$ while the other travels at $132\ \frac{f}{s}$, then we can express the distance *d* feet between the trains as a function of *t* seconds after noon.
$d = 5280\,f - 242\ \frac{f}{s}\ t$.

■ Exercise 1

[1] A rocket burns fuel at the rate of $2000\ \frac{\text{kg}}{\text{min}}$. Write the mass *m* of the fuel burned as a function of time *t* minutes.

[2] A bottling company can fill 2000 bottles of juice per hour. Write the number *n* bottles filled as a function of *t* hours.

[3] A paving company surfaces roadway at a constant rate of 12 miles per day. Write the number of *y* miles surfaced as a function of time *t* hours.

[4] The fuel tank of a certain car contains 20 gallons of gasoline. Suppose the car gets 30 miles to the gallon of fuel when traveling on a straight and level road at 60 miles per hour. Write the volume *V* of fuel in the tank as a function of time *t* hours traveled on a straight and level road.

In all of the examples above, a rule could be written showing how to compute one number given another. Moreover, each rule was of a kind that for the given number exactly one number would result from the computation.

EXAMPLE 7. Suppose a bakery sells donuts for 60¢ each. Then the gross profit *$P* from selling *n* number of donuts is $P = 0.60\ \frac{\$}{\text{donut}} \times n$. How much money will the bakery take in from selling (a) 200 donuts? (b) 317 donuts?

(a) $P = 0.60\ \frac{\$}{\text{donut}} \times n$.
When $n = 200$ donuts, $P = 0.60\ \frac{\$}{\cancel{\text{donut}}} \times 200\ \cancel{\text{donuts}} = P = \$0.60 \times 200 = \$120$.

(b) $P = 0.60\ \frac{\$}{\text{donut}} \times n$.
When $n = 317$ donuts, $P = 0.60\ \frac{\$}{\cancel{\text{donut}}} \times 317\ \cancel{\text{donuts}} = P = \$0.60 \times 317 = \$190.20$

EXAMPLE 8. A rocket burns fuel at the rate of $2000\ \frac{\text{kg}}{\text{min}}$. Write the mass *m* of the fuel burned as a function of time *t* minutes. How much fuel will have been used at exactly 1 minute and 15 seconds after launch?

A function of Example 8 allows us to know, by merely performing a sequence of arithmetic computations, the exact amount of fuel used at any time after launch. And, there is no ambiguity about the amount of fuel used. The function predicts exactly one amount of fuel used in a given amount of time.

PROBLEM 1. A paving company surfaces roadway at a constant rate of 12 miles per day. Write the number of *y* miles surfaced as a function of time *t* hours. Make a table showing the distance surfaced at 0 hr, 0.5 hr, 1.0 hr, 1.5 hr and so on up to 4 hours.

In order to convey the idea that a function pairs one number with another, we use

Answers

■ Exercise 1

[1] $m = 2000\ \frac{\text{kg}}{\text{min}} \cdot t$.

[2] $n = 2000\ \frac{\text{bottles}}{\text{hr}} \cdot t$.

[3] $y = 0.5\ \frac{\text{miles}}{\text{hr}} \cdot t$

[4] $V = 2\ \frac{\text{gal}}{\text{hr}} \cdot t$

PROBLEM 1. $y = 0.5\ \frac{\text{miles}}{\text{hr}} \cdot t$

t (hr)	0	.5	1	1.5	2	2.5	3	3.5	4
y (mi)	0	.25	.5	.75	1	1.25	1.5	1.75	2