Exercises [A]

1. What number serves as the identity element for multiplication in the set of integers?
2. Does the set of integers contain the inverse of each element under the operation of multiplication? Does the set contain the inverse of any of its elements under multiplication?
3. Solve the equation $-4 + x = -13$ in the set of integers. Does the equation $-4 \cdot x = -13$ have a solution in I?
4. A mathematical system is given by $\{0, 1, 2, 3\}$ under "addition" $(\oplus)$ with the understanding that the "sum" of a and b is the remainder when $a + b$ is divided by 4. The addition table is shown.

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

   In this system:
   (a) Which is the identity element?
   (b) Write the inverses of 1 and 2.
   (c) Solve the equation $2 \oplus x = 1$.
5. (a) Make the table for the system $\{0, 1, 2, 3, 4\}$ under "addition" $(\oplus)$ with the understanding that the "sum" of a and b is the remainder when $a + b$ is divided by 5.
   (b) Write the inverses of 3 and 4 in this system.
   (c) Solve the equation $3 \oplus x = 2$ in this system.
6. (a) Make the table for the system $\{0, 1, 2, 3, 4, 5\}$ under "addition" $(\oplus)$ if the "sum" of a and b is the remainder when $a + b$ is divided by 6.
   (b) Write the inverses of 1, 2, 3 in this system.
   (c) Solve the equation $4 \oplus x = 2$ in this system.
7. In the system consisting of $\{a, b, c\}$ with $\circ$ as defined by the table:

$\circ$	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

   (a) Select the identity element.
   (b) List each element with its inverse.
   (c) Solve $x \circ b = c$.
8. In the system consisting of $\{a, b, c, d\}$ with $\otimes$ as defined by the table:

$\otimes$	a	b	c	d
a	d	c	b	a
b	c	a	d	b
c	b	d	a	c
d	a	b	c	d

   (a) Select the identity element.
   (b) List each element with its inverse.
   (c) Solve $b \otimes x = d$.
9. (a) Make a table for the system $\{0, 1, 2, 3\}$ under "multiplication" $(\otimes)$ with the understanding that the "product" of a and b is the remainder when ab is divided by 4.
   (b) What is the identity element in this system?
   (c) Does 0 have an inverse in this system?
   (d) What is the inverse of 3 in this system?
   (e) Does 2 have an inverse in this system?
   (f) Is there a solution of $2 \otimes x = 1$ in this system?

10. (a) As in Exercise 9, make a table for the system $\{0, 1, 2, 3, 4\}$ under "multiplication" ($\otimes$) if the "product" of a and b is the remainder when ab is divided by 5.

(b) In this system, write each non-zero element with its inverse.

(c) Solve $2 \otimes x = 1$ in this system.

11. (a) As in Exercise 9, make a table for the system $\{0, 1, 2, 3, 4, 5\}$ under "multiplication" ($\otimes$) using the remainder when ab is divided by 6 as the "product."

(b) Which of the non-zero elements have inverses in this system?

(c) In this system, find the solution set of $3 \otimes x = 3$.

(d) In this system, find the solution set of $3 \otimes x = 2$.