

Consider the following “proofs”.

$$\begin{aligned}1 & \\ &= (-1)(-1) \\ &= \sqrt{(-1)(-1)} \\ &= \sqrt{-1} \sqrt{-1} \\ &= i \cdot i \\ &= i^2 \\ &= -1\end{aligned}$$

$$\therefore 1 = -1$$

What went wrong?

$$\begin{aligned}4 &= (-2)(-2) \\ \sqrt{4} &= \sqrt{(-2)(-2)} \\ \sqrt{4} &= \sqrt{-2} \sqrt{-2} \\ \sqrt{4} &= i\sqrt{2} \cdot i\sqrt{2} \\ \sqrt{4} &= i^2 \sqrt{2} \cdot \sqrt{2} \\ \sqrt{4} &= 2i^2 \\ 2 &= -2\end{aligned}$$

$$\therefore 2 = -2$$

What went wrong?

A copy of the laws for manipulating square roots is attached to this.