[11-01-27-RT]

Sums of trig functions

■ **Write each function in the form $y = a \sin k(x - \beta)$ and state the maximum value the function attains.**

[1] $f(x) = \sin x + \cos x$

[2] $f(x) = \sin x - \cos x$

[3] $f(x) = 4 \sin x + 3 \cos x$

[4] $f(x) = \sin 2x + 3 \cos 2x$

■ **Write each function in the form $A \sin x + B \cos x$**

[5] $f(x) = \sin\left(x - \frac{\pi}{4}\right)$

[6] $f(x) = 4 \sin\left(x + \frac{\pi}{6}\right)$

Answers

■ **Write each function in the form $y = a \sin k(x - \beta)$ and state the maximum value the function attains.**

[1] $f(x) = \sin x + \cos x = \sqrt{2}\left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} \sin x + \frac{\sqrt{2}}{2} \cos x\right) =$

$\sqrt{2}\ (\cos\phi \sin x + \sin\phi \cos x) = \sqrt{2}\ \sin(x + \phi),\ \phi = \frac{\pi}{4}\Big),\ f_{\max} = \sqrt{2}$

[2] $f(x) = \sin x - \cos x = \sqrt{2}\left(\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x\right) =$

$\sqrt{2}\ (\cos\phi \sin x - \sin\phi \cos x) = \sqrt{2}\ \sin(x - \phi),\ \phi = \frac{\pi}{4}\Big),\ f_{\max} = \sqrt{2}$

[3]
$f(x) =$

$4 \sin x + 3 \cos x = \sqrt{5}\left(\frac{4}{\sqrt{5}} \sin x + \frac{3}{\sqrt{5}} \cos x\right) = \sqrt{5}\ (\cos\phi \sin x + \sin\phi \cos x) = \sqrt{5}\ \sin(x + \phi),$

$\phi \approx 0.6435\Big),\ f_{\max} = \sqrt{5}$

Note: $\left(\frac{4}{x}\right)^2 + \left(\frac{3}{x}\right)^2 = 1 \Longrightarrow \frac{25}{x^2} = 1 \Rightarrow x = \sqrt{5}$

[4]
$f(x) = \sin 2x + 3 \cos 2x = \sqrt{10}\left(\frac{1}{\sqrt{10}} \sin 2x + \frac{3}{\sqrt{10}} \cos 2x\right) = \sqrt{10}\ (\cos\phi \sin 2x + \sin\phi \cos 2x) =$

$\sqrt{10}\ \sin(2x + \phi) = \sqrt{10}\ \sin 2\left(x + \frac{\phi}{2}\right),\ \frac{\phi}{2} \approx 1.2491\Big),\ f_{\max} = \sqrt{10}$

You might see this anwer written like: $\sqrt{10}\ \sin 2(x + \theta),\ \theta \approx 2.49811$

■ **Write each function in the form $A \sin x + B \cos x$**

[5] $f(x) = \sin\left(x - \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} - \sin\frac{\pi}{4} \cos x = \frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x$

[6]
$f(x) = 4 \sin\left(x + \frac{\pi}{6}\right) = 4\left(\sin x \cos\frac{\pi}{4} + \sin\frac{\pi}{4} \cos x\right) = 4\left(\frac{\sqrt{2}}{2} \sin x + \frac{\sqrt{2}}{2} \cos x\right) = 2\sqrt{2}\ (\sin x + \cos x)$