Derivative of $\exp(x)$ or equiv $\frac{d}{dx} e^x$

$y = e^x$ get $\frac{dy}{dx}$

$\ln y = \ln(e^x)$

$\ln y = x$

$\frac{d}{dx} \ln y \quad \frac{dy}{dx} = \frac{d}{dx} x$, y is fn of x

CHAIN RULE

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

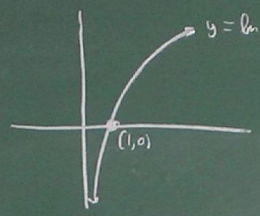
$\frac{1}{y} \frac{dy}{dx} = 1$

$\frac{dy}{dx} = y$

$\frac{dy}{dx} = e^x$

$$\frac{d}{dx} e^x = e^x$$

[7.3] Exponential function



$y = \ln x$, strictly increase

$\frac{dy}{dx} = \frac{1}{x} > 0$

$\Rightarrow \ln x$ 1-1

$\Leftrightarrow \ln x$ invertible

Find the inverse fn.

$y = \ln x$ invertible

call its inverse "the exponential fn"

$y = \exp(x)$

$y = \exp x$

Infer,

$y = \ln x \equiv x = \exp y$

$y = \exp x \equiv x = \ln y$

DMN of $\exp(x)$ is $\forall x \in \mathbb{R}$

RNG of $\exp(x)$ is $y > 0$

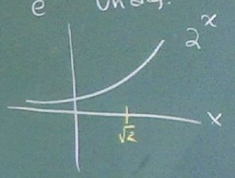
$\exp(\ln x) = x$, $x > 0$

$\ln(\exp x) = x$, $x \in \mathbb{R}$

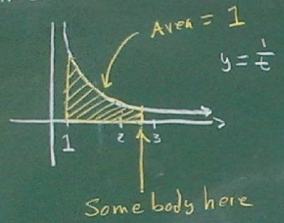
$\exp(0) = 1$, $\because 0 = \ln 1$

$e^{\sqrt{2}}$ define

$e^{\sqrt{2}}$ undef.



Meet e



e

$$\int_1^e \frac{dt}{t} = 1$$

$\ln e = 1$

No news, for r rational

$$e^r = \exp(\ln e^r)$$
$$= \exp(r \ln e)$$
$$= \exp(r)$$

$\therefore$ r rational

Inv fn

$\therefore \ln e = 1$

$$\boxed{e^r = \exp(r)}$$

Idea ...

Definition
$e^x = \exp(x)$ for $x \in \mathbb{R}$

Hope that w/ this def, exponent Laws hold

Prove Exp Laws

Prove $e^a e^b = e^{a+b}$, $a, b \in \mathbb{R}$

Proof

$$e^a e^b = \exp(\ln e^a e^b)$$
$$= \exp(\ln e^a + \ln e^b)$$
$$= \exp(a \ln e + b \ln e)$$
$$= \exp(a+b)$$
$$= e^{a+b}$$

□

① $\frac{d}{dx} e^{\sqrt{x}} = e^{\sqrt{x}} \frac{d}{dx}\sqrt{x}$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

② $\frac{d}{dx} e^{x^2 \ln x} = e^{x^2 \ln x}\left[2x \ln x + \frac{x^2}{x}\right]$

$$= x e^{x^2 \ln x}(2 \ln x + 1)$$
$$= x e^{x^2 \ln x}(1 + \ln x^2)$$